

Hot Markets, Investor Sentiment, and IPO Pricing

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Abstract

We develop a model of the IPO process that ties three IPO ‘anomalies’ – underpricing, hot markets and long-run underperformance – to demand from an evolving class of exuberant investors. While long-run underperformance is often attributed to a decline in investor sentiment, the more novel issue we investigate is whether underpricing and other patterns in hot markets are also explained by investor sentiment. We resolve the apparent paradox that issuers, while shrewdly timing their IPOs to take advantage of optimistic valuations, are less than aggressive about pricing their offerings. IPO issuer proceeds are maximized by a policy that allocates stock to regular (institutional) investors at a discount. These shares are resold to sentiment investors in stages, at prices regular investors maintain by restricting share availability. Underpricing compensates regular investors for expected losses in the event of the hot market ending prematurely. The model is consistent with many institutional aspects of the IPO process and generates several new empirical predictions.

1 Introduction

There are three anomalous aspects to the process by which firms go public. Initial public offerings (IPOs) appear to be substantially ‘underpriced’ and exhibit positive first-day returns on average. More disturbing to the notion of market efficiency, however, is the finding that the initial price run-up appears to be undone in subsequent months as IPO stocks underperform stocks of matched firms.¹ Hence, from the vantage of a longer horizon, IPOs can arguably be regarded as ‘overpriced’ in the after-market. The strength of these patterns varies over time, with both the initial price run-up and subsequent underperformance more dramatic in ‘hot’ periods of high IPO volume [Ritter (1984, 1991)].

What is one to make of these patterns? The literature offers no consensus. Numerous articles model the initial underpricing, an empirical fact that was recognized and studied well before work on longer-term price patterns emerged. For the most part these models, based as they are on investor rationality in incomplete information settings, have shed little light on the long-run performance of IPOs.² In contrast, work on long-run performance is primarily empirical and emphasizes the role of investor sentiment and bounded rationality in explaining the price behavior of IPO stocks. The impact of investor sentiment is regarded as particularly acute in hot markets. Over time, investor exuberance fades, resulting in long-run underperformance. Loughran, Ritter, and Rydqvist (1994) go further in claiming that issuers ‘time’ their IPOs to coincide with periods of excessive optimism. Such patterns can persist if rational investors are dissuaded by the cost of implementing arbitrage strategies [Shleifer and Vishny (1997)].

The behavioral story sketched above has some obvious appeal, but it raises an apparent paradox: if issuers are regarded as rational and shrewd enough to choose a hot market in which to go public, why are they less than aggressive in setting the offer price? After all, it seems plausible that the presence of sentiment investors could lead to *higher* offer prices and a *lower*

¹See Ritter (1991), Loughran and Ritter (1995), and others. We assume the existence of IPO underperformance notwithstanding recent papers that have questioned its nature and significance [Brav, Geczy, and Gompers (2000), Eckbo and Norli (2001)].

²Among these are explanations based on the ‘winner’s curse’ [Rock (1986)], signaling [Allen and Faulhaber (1989), Welch (1989)], cascades [Welch (1992)], and investor incentives to reveal information truthfully [Benveniste and Spindt (1989)].

level of underpricing as rational issuers take advantage of them. Reconciling the simultaneous existence of underpricing and long-run underperformance thus requires additional structure on the behavioral assumptions and the nature of the economic environment.

The task we set ourselves in the paper is, therefore, to develop a model of IPO pricing in hot issue markets that elucidates the connection between underpricing and long-run underperformance. We ask, what should a profit-maximizing issuer do in the presence of exuberant investor demand and short sale constraints? We argue that the issuer should seek to capture as much as possible of the surplus under the exuberant investors' demand curve, in a setting where demand may build over time. We derive an optimal mechanism (which we argue is consistent with institutional reality) that achieves the issuer's first-best outcome. In so doing, we also employ the model to shed light on some of the 'stylized' institutional features of the IPO process. Salient among these are allocation practices that discriminate against retail investors, underwriters' price support activities, penalty bids on syndicate members whose clients 'flip' the stock in the early after-market, and lock-up provisions barring insiders from selling shares in the first 180 days [Jenkinson and Ljungqvist (2001)].

The model starts with the premise that some investors may, on occasion, be 'irrationally exuberant' about the prospects of IPOs from, say, a particular industry. Assuming constraints on short sales, this is consistent with the presence of long-run IPO underperformance.³ More interestingly, the model suggests possible connections between IPO underperformance and the initial price run-up. We show that value to an issuer is maximized by underwriters allocating IPO shares to their regular (institutional) investors for gradual sale to sentiment investors as they arrive in the market over time. Regulars maintain IPO stock prices – thereby extracting surplus from sentiment investors – by holding stock inventory and restricting the availability of shares. Underpricing emerges as a fair compensation to the regulars for expected inventory losses arising from the possibility that sentiment demand may cease. In return, the expo-

³In a different setting Miller (1977) shows that a divergence of beliefs – similar to the notion that some investors are more optimistic than others – can lead to long-run IPO underperformance.

Brav and Gompers (1997) show empirically that underperformance is concentrated amongst small, non-venture backed companies. These are the sorts of companies whose stock is more likely to be owned by individuals, who in turn may be more easily influenced by fads or lack of complete information. Arguably, stocks of smaller companies are also more expensive to short than stocks of larger companies, which may account for the persistence of poor long-run returns.

priation of value from sentiment investors is capitalized into a higher offer price than would otherwise be the case.

For the inventory holding strategy to be implemented, there must either be a dominant investor or, when there are many investors, it must be incentive compatible for regular investors not to deviate by selling their IPO allocations prematurely. To deter cheating, it may be necessary for the underwriter to punish deviations from the equilibrium strategy. We show that the degree of the underwriter's ability to impose penalties determines the optimal size of an offer, the extent of underpricing, and subsequent long-run performance.

It is worth emphasizing that when there is a dominant investor or when the underwriter can impose sufficient costs to ensure cooperation among regular investors, the full benefits are passed on to the issuer in the form of a higher offer price. In the economic environment we model, issuers cannot do better by the use of alternative ways to sell equity. For instance, if the issuer were to engage in a quick succession of equity offerings (an IPO followed by seasoned offerings), the value obtained would not exceed the value from the inventory holding process we model. In any case, issuing stock repeatedly over a short period is implausible, given significant economies of scale in issuing costs and the necessity to satisfy registration and disclosure requirements repeatedly.

Our paper has a focus quite different from much of the existing work in behavioral finance. The behavioral finance literature has tended to focus on asset pricing anomalies, such as the predictability of returns, the equity premium puzzle, and under- and over-reactions [for an exhaustive survey, see Hirshleifer (2001)]. Our model is an attempt to capture the equilibrium response of issuers and underwriters in the face of divergence of opinion among investors. It is thus related to an empirical literature in which firms act strategically to take advantage of the market's mispricing or mis-perceptions.⁴

We do not attempt to rationalize the existence or behavior of exuberant investors. Biases that might lead to such behavior have been studied by psychologists for some time and financial economists have recently introduced them into formal models of asset pricing. For example,

⁴For example, see D'Mello and Shroff (2000) and Dittmar (2000) on firms' strategic use of share repurchases.

a large literature reports that people believe their knowledge to be more accurate than it really is [for a review, see Odean (1998)]. Overconfidence can persist if economic agents do not appropriately learn from outcomes, which may be due to another bias, ‘self-attribution’. Experiments have shown that people tend to attribute favorable outcomes to their abilities and unfavorable ones to chance or other external factors beyond their control [see Daniel, Hirshleifer, and Subrahmanyam (1998) for a discussion of the psychology literature].⁵

The test of a model that relies on investor sentiment is the power of its refutable empirical predictions. Our model generates a number of novel predictions:

- As investor sentiment grows, IPO offer sizes increase. Lower-quality companies are taken public, resulting in a decrease in average issuer quality. Companies become more likely to raise money for non-investment purposes, such as paying down debt.
- Underwriters penalize investors who engage in excessive flipping. Importantly, they do so even in IPOs that do not receive price support. Such penalties are targeted primarily at retail and infrequent investors.
- Corporate insiders are released early from their lock-up provisions, if after-market demand from sentiment investors is unexpectedly high, once regular investors have unloaded their excess inventory, or if the hot market has come to an end.

Our model also addresses several hitherto puzzling empirical findings:

- Ritter (1991) documents that underpricing and long-run performance are negatively related, while Krigman, Shaw, and Womack (1999) find a positive relation. Our model shows that the relation is not necessarily monotonic. In particular, we show that the relation is negative only if the probability of the hot market ending is small.
- Loughran and Ritter (2002) report evidence that the offer price is not fully revised relative to the filing range in response to *public* information that emerges during the bookbuilding

⁵Daniel et al. (1998) combine the two biases to show that a favorable initial shock to private information causes the price to rise beyond the unbiased value. Accumulating evidence eventually forces investors to a more reasonable self perception. This leads to positive short-lag correlations and negative long-lag correlations. Our sentiment investors could possibly be going through similar cycles. In a related paper, Gervais and Odean (2001) analytically model the learning process under self-attribution bias.

phase, and argue this contradicts Benveniste and Spindt's (1989) private information revelation model. In our model, the 'partial adjustment' of the offer price is driven by the difference in opinion between rational and sentiment investors and not by private information. Thus, unlike Benveniste and Spindt (1989), our model can accommodate partial adjustment in response to public stock price movements.

- The empirical evidence on the relation between underwriter prestige and underpricing is mixed. Consistent with evidence from the 1990s, we predict that underpricing *increases* in underwriter prestige, but that this relation depends on the state of the IPO market.

Two recent papers that test some of the main predictions of our model, and that provide strong empirical support for it in the context of the recent 'dotcom mania', are Ofek and Richardson (2002) and Dorn (2002). Ofek and Richardson show that high initial returns occur when institutions sell IPO shares to retail investors on the first day, and that such high initial returns are followed by sizeable reversals to the end of 2000, when the bubble had burst. This is precisely the pattern we predict, and it highlights the importance of heterogeneous beliefs and short sale constraints in explaining both the initial IPO price run-up and longer-term performance. Using German data on IPO trading by 5,000 retail customers of an online broker, Dorn documents that retail investors overpay for IPOs following periods of high underpricing in recent IPOs, and for IPOs that are in the news. Consistent with our model, he also shows that hot IPOs pass from institutional into retail hands. Over time, high initial returns are reversed as net purchases by retail investors subside, eventually resulting in underperformance over the first six to 12 months after the IPO.

The paper proceeds as follows. The basic model is developed in Section 2. In Section 3, we analyze the issuer's optimal unconstrained strategy for extracting surplus from the exuberant investors. Since this strategy would violate regulatory rules, we derive in Section 4 an alternative mechanism that implements the optimal strategy, which involves inventory-holding by a regular investor. We solve for the optimal issue size and offer price, and derive the patterns of prices in the short- and long-run. We also analyze the comparative statics of the price patterns with respect to the strength of sentiment demand and the probability of the hot market coming

to an end. Section 5 considers two extensions to the model: multi-period sentiment demand, and multiple regular investors. In Section 6, we discuss empirical support for various aspects of the model and offer new testable implications. Concluding remarks are in Section 7.

2 The Model

We model a firm that is going public in a ‘hot’ IPO market, to be defined shortly. The firm’s equity is sold via a standard firm-commitment IPO in which an underwriter (or underwriting syndicate) assumes responsibility for distributing the issuer’s shares to investors. The offer price in such IPOs is usually finalized at the end of bookbuilding, just prior to the start of trading. The offer is subject to a uniform-pricing rule such that neither the issuer nor the underwriter can price-discriminate among investors [see also Benveniste and Wilhelm (1990)]. The offer size Q and price P_0 will be chosen so as to maximize the owner-manager’s wealth.

The demand side of the IPO market consists of two types of investors. The first type are small, unsophisticated investors who are infrequent participants in the stock market. These investors are prone to episodes of optimistic or pessimistic ‘sentiment’ about the stock market, especially IPOs, where sentiment denotes incorrect beliefs about the fundamental value of an asset arising from treating noise as relevant information [Black (1986)]. We will label these investors sentiment or ‘s-type’ investors. In our set-up, a ‘hot’ IPO market is one characterized by the presence of optimistic investors.⁶ Pessimistic investors, if present, are prevented from expressing their demands by short sale constraints, which are pervasive in IPOs.⁷ As discussed

⁶This mirrors Miller’s (1977) divergence-of-opinion model. Our sentiment investors hold beliefs that are in the right tail of the distribution of beliefs. Their beliefs might, for instance, be driven by a ‘halo effect’ [Nisbett and Wilson (1977)]. The halo effect causes an individual to extend a favorable evaluation of one characteristic to other characteristics. For example, a favorable evaluation of a firm’s product might be extended to its expected future stock returns, or investors might extend positive news about a firm’s accounting earnings to its stock returns [see Lakonishok, Shleifer and Vishny (1994), Shefrin and Statman (1995)].

⁷Geczy, Musto, and Reed (2002) show that borrowing IPO stock in the early after-market is extremely expensive in general, the more so, the higher was the initial day return. Houge, Loughran, Suchanek, and Yan (2001) discuss some of the reasons why this may be. First, brokers can only allow clients to short-sell if delivery of the borrowed shares can be guaranteed, which effectively rules out short sales in the first few days as share allocations are not distributed immediately. Second, short sellers face difficulty borrowing stock as regulations and market practices restrict the potential supply. Stock could in principle be borrowed from corporate insiders, syndicate banks, or investors who hold shares in the aftermarket. However, insiders in most IPOs are ‘locked-up’ for some period of time following the IPO, usually 180 days, which prevents them from selling or lending their shares. Banks in the IPO syndicate are prohibited by the SEC from lending shares in the first 30 days of trading. And most IPOs involve such a small part of the equity that the ‘free float’ in public hands tends to be very small.

later, though they hold excessively optimistic beliefs about the prospects of firms going public, s -type investors still act rationally given their beliefs.

The second type of investor holds beliefs that correspond to an unbiased estimate of the issuing firm's future prospects. It may be reasonable, for instance, to regard institutional investors as belonging to this category. For expositional ease, we will label these investors 'rational'. All other market participants (issuers, underwriters) are taken to be rational and value-maximizing as well. There is no private or asymmetric information in the model, and the nature and characteristics of the market participants and their beliefs are common knowledge. Hence, sentiment and rational investors know each others' beliefs, but still 'agree to disagree' on the valuation of the IPO shares.⁸ For simplicity, everyone is taken to be risk-neutral.

The model has four relevant dates: $t = 0, 1, 2$, and T . At $t = 0$, the period prior to the offering, the pricing and size of the IPO are determined. Date $t = 1$ is the initial day of trading in the IPO shares. Once trading has begun, the market may continue to be hot – that is, it may continue to be characterized by the presence of optimistic investors – but sooner or later the hot market will come to an end. This captures the notion that there will eventually be incontrovertible evidence of the IPO shares being overpriced, or that the cost of shorting IPO stock will fall to the point where prices are no longer set by optimistic investors.⁹ For now, we model this by introducing a subsequent trading date $t = 2$ at which the IPO market may or may not still be hot. In Section 5.1, we will explicitly extend the model to a multiple-period setting. Finally, T is the terminal date by which the hot market is definitely over and there is no more disagreement about firm value.

We denote by γ the (exogenous) probability of the hot market ending at $t = 2$. In addition to disagreeing about value, investors disagree about γ . Rational investors understand that the hot market may end before T with probability $\gamma > 0$, in which case the marginal investor will be someone holding unbiased beliefs. Sentiment investors, on the other hand, dismiss this possibility: in their mind, the hot market will continue for sure.¹⁰

⁸The notion of investors 'agreeing to disagree' is commonly employed in models with a diversity of opinions among market participants, a good example being Harris and Raviv (1993).

⁹Ofek and Richardson (2002) show that the bursting of the dot-com bubble in March/April 2002 coincided with a substantial increase in the availability of stock to borrow.

¹⁰To avoid problems with Bayesian updating from a zero probability prior, it is easiest to assume that

Let V_T denote the terminal payoff of the security at T . There are no distributions (*e.g.* dividends) and the discount rate is zero. At $t = 1$, the ‘fundamental’ or long-term expected value of an IPO share – the value from the perspective of rational investors – is denoted by $V_R = E(V_T)$. Absent sentiment investors and additional information, V_R would be the market price of the IPO shares at $t = 1$. As we will discuss, the presence of sentiment investors, with their more optimistic valuations, can affect the pattern of market prices of IPO shares. The question of interest to us is not so much the existence of sentiment *per se*, but rather the impact optimistic investors may have on pricing and trading patterns, and thus on the institutional arrangements that result.

The value sentiment investors place on the IPO shares is not uniform. Specifically, we assume that sentiment investors are budget-constrained and that their aggregate demand curve for IPO shares can be represented as

$$V_s = V_R + a - \lambda Q \tag{1}$$

where Q is the total number of IPO shares held by sentiment investors. Define $\bar{Q} = \frac{a}{\lambda}$. For all $Q < \bar{Q}$, s -types (if they are present) place a value higher than V_R on the IPO shares. Sentiment investors know the demand curve and the value put on the shares by rational investors.

We view the sentiment investors as infrequent traders, many of whom may not be active in the market at any particular time. If the hot market continues at $t = 2$, additional s -type investors may arrive in the market.¹¹ Thus, sentiment demand can evolve over the two periods (or, in Section 5.1, over multiple periods). The fact that sentiment demand can build over time affects the interpretation of demand in equation (1). Acting rationally, the sentiment investors present in the market at $t = 1$ would never be willing to pay a price at $t = 1$ that is greater than the expected price conditional on their beliefs at $t = 2$. Thus, they properly anticipate the prices of the security in the short run, by forecasting what demand will be at $t = 2$.

Conditional on the (mistaken) belief that the hot market will continue at $t = 2$ for certain,

sentiment investors do have a prior non-zero, but infinitesimally small, probability of such a revision of their valuation. This does not affect any of the discussion and we will ignore this infinitesimal probability in the expressions.

¹¹The fact that not all sentiment investors are present in the market at $t = 1$ may be the result of the time required for information to disseminate among the less informed investors; for enthusiasm about the IPO to build while the market stays hot; or the additional time needed for some sentiment investors to raise resources and bid for IPO shares, especially when many ‘hot’ IPOs come to market around the same time.

s -type investors expect demand at $t = 2$ to be the aggregate demands of s -types arriving at $t = 1$ and $t = 2$. It is this ‘longer-term’ demand (and not just the $t = 1$ sentiment demand alone) that affects the value V_s they put on the IPO shares in equation (1). Note here that we are maintaining the assumption that no short positions in the IPO stock are possible (or that they are simply too expensive to execute).

We can now determine the price of the IPO shares at $t = 2$ and, thereby, the offer and trading prices at $t = 0$ and 1. If the hot market has ended, the price at $t = 2$ will be set by the expectations of the rational investors such that $P_2 = V_R$. If the hot market persists, the price will be given by the demand curve in equation (1). We assume here that the quantity of shares sold is such that $Q < \bar{Q}$. This, as we show later, is consistent with an optimal choice for Q . For a given quantity of shares, Q , issued at $t = 0$, the valuations by the rational and s -type investors are determined by their beliefs as follows:

- Rational investors: $E^R(P_2) = \gamma V_R + (1 - \gamma)E^s(P_2)$
- Sentiment investors: $E^s(P_2) = V_R + a - \lambda Q$

The expected values above represent the prices that s -type and rational investors should be willing to pay at $t = 1$, given their beliefs regarding P_2 . Note that the rational investors’ valuation $E^R(P_2)$ is greater than their long-run valuation V_R , since they expect to be able to sell the security to s -types at $t = 2$ with probability $(1 - \gamma)$.

To summarize, we model a ‘hot’ market that is characterized by the presence of optimistic investors. Not all optimistic investors are present at $t = 1$ and, if the hot market persists, more are expected to show up at $t = 2$. Rational investors expect the terminal value of the IPO shares to be V_R . Unlike the optimists, they recognize that the hot market may come to an early end at $t = 2$, with probability $\gamma > 0$. In the longer run, by the terminal date T , the hot market will end with certainty. All investors, rational or otherwise, act in a manner consistent with their beliefs.

3 Selling IPO Shares

We consider the optimal procedure for selling IPO shares so as to maximize issuer wealth in the presence of optimistic valuations. For now we maintain the assumption that the offer quantity Q is given exogenously. The unconstrained optimum involves selling of IPO shares by underwriters at both $t = 1$ and $t = 2$. This is, of course, contrary to the market practice of selling the shares in a single shot and the requirement that investors be sold IPO shares at a uniform price. As we will see, such discretion will have no impact if *all* the demand from s -type investors is present at $t = 1$ or if the demand by s -type investors at $t = 1$ is large enough to absorb the full offering.

Let q_1 be the number of IPO shares sold at $t = 1$, while q_2 is sold at $t = 2$. The expected proceeds, Ψ , to the issuer are

$$\Psi = q_1 P_1 + q_2 (E^s(P_2) (1 - \gamma) + V_R \gamma).$$

Given their beliefs, sentiment investors expect the price at $t = 2$ to be $E^s(P_2) = V_R + a - \lambda(q_1 + q_2)$. Hence, so long as the sentiment investors hold all the IPO shares issued at $t = 1$, the marginal investor is a sentiment investor and the price at $t = 1$ will be $E^s(P_2)$. Let $\bar{Q}_1 \leq \bar{Q}$ denote the total optimistic demand present at $t = 1$. If $q_1 > \bar{Q}_1$, the marginal investor is a rational investor and the price at $t = 1$ will be $E^R(P_2)$. Thus we have:

$$P_1 = \begin{cases} V_R + a - \lambda(q_1 + q_2) & \text{if } q_1 \leq \bar{Q}_1 \\ \gamma V_R + (1 - \gamma)(V_R + a - \lambda(q_1 + q_2)) & \text{if } q_1 > \bar{Q}_1 \end{cases}. \quad (2)$$

Assuming the firm does not need to raise a particular level of financing, the owner-manager's objective is simply to maximize the 'profit' from selling IPO shares. That is, his objective is to maximize the excess value, denoted by Π , of the proceeds over his own valuation $V_R Q$. The optimal (q_1^*, q_2^*) can, therefore, be regarded as the solution to the following constrained optimization problem:

$$\begin{aligned} \max_{q_1, q_2} \Pi &\equiv \Psi - V_R Q = q_1 P_1 + q_2 (E^s(P_2) (1 - \gamma) + V_R \gamma) - V_R Q \\ \text{s.t. } & q_1 + q_2 = Q \end{aligned}$$

Its solution is given in the following proposition.

Proposition 1 *For a given number of shares to be issued, Q , the optimal choice of q_1^* and q_2^* is such that*

$$(q_1^*, q_2^*) = \begin{cases} (Q, 0) & \text{if } Q \leq \bar{Q}_1 \\ (\bar{Q}_1, Q - \bar{Q}_1) & \text{if } Q > \bar{Q}_1 \end{cases} . \quad (3)$$

Proof. See the appendix. ■

Proposition 1 shows that the issuer may do better by staggering the sale of equity over two time periods instead of one. By restricting the initial supply of shares, the issuer ensures that the marginal investor at $t = 1$ is a sentiment investor. If, however, the total quantity Q to be sold is less than the demand by sentiment investors at $t = 1$, then the issuer optimally chooses to set q_2^* equal to zero. The intuition is straightforward. In our set-up there is no price advantage from delaying the sale of equity if it can be sold to sentiment investors at $t = 1$. Delay exposes the issuer to the risk of the market crashing at $t = 2$. Hence, the issuer is strictly better off selling to the sentiment investors at $t = 1$ and thus taking advantage of their mistaken belief that the hot market will persist at $t = 2$. As we will discuss later, a similar result holds when the model is extended to consider the arrival of sentiment investors over a larger number of periods.

Proposition 1 indicates that it may be optimal to sell an offering in stages. However, as mentioned, laws and regulations effectively prevent issuers and their underwriters from conducting firm commitment offerings in a staggered fashion. In the U.S., for instance, NASD rule IM-2110-1 on “Free-riding and Withholding” prevents an underwriter who holds IPO shares in inventory from selling them in the after-market above the offer price.¹² Thus, there is considerable downside risk without upside potential. We now consider an alternative arrangement by which an underwriter can achieve the same ends without directly selling the IPO in stages.

¹²Countries where staggered sales are possible include Germany. Though rare and usually confined to small companies, such offerings proceed as follows. Rather than allocating stock to investors at $t = 0$, the issuer announces a quantity Q it intends to sell via the stock exchange, in one or more trading sessions, at the market-clearing price. This closely resembles our mechanism.

4 Inventory Holding by Institutional Investors

Given constraints on the underwriter's ability to (directly) stagger the sale, we suggest that institutional (or other 'regular') investors can be delegated the task of holding inventory in the after-market for resale to sentiment investors. Specifically, we assume (for now) that there exists a monopolist regular investor who purchases Q shares at the offer price P_0 and then sells q_1 shares at $t = 1$ and the remainder q_2 at $t = 2$, when the full demand by s -type investors is established (so long as the hot market persists). The assumption of a single (or dominant) regular investor simplifies the exposition and abstracts from concerns about free-riding among regular investors. The case with a multitude of regular investors is discussed later, with the threat of punishment dissuading regulars from engaging in free-riding behavior.

Once the shares have been allocated, the regular investor's problem is no different from that of the issuer. Thus, the regular investor will find it optimal to follow the staggered sale strategy, where the aggregate quantities sold in the secondary market at $t = 1$ and $t = 2$ are given by q_1^* and q_2^* , respectively. The staggered sale strategy requires the regular investor to hold q_2^* shares in inventory from $t = 1$ to $t = 2$, when the quantity to be sold is such that $Q > \bar{Q}_1$. Given our assumption of a monopolist profit-maximizing regular investor, there is no incentive to deviate by selling the shares early.

4.1 Optimizing Offer Size and Price

In equilibrium, a regular investor will invest in IPOs only if he does not expect to lose as a consequence. If an IPO share is bought at an offer price P_0 , the regular investor's participation constraint can be stated as

$$-QP_0 + q_1^*P_1 + q_2^*[(1 - \gamma)E^s(P_2) + \gamma V_R] \geq 0 \quad (4)$$

where q_1^* and q_2^* are as given in (3). In the above equation, the first term is the cost of purchasing all the shares in the IPO. The second and third terms represent the cash flows received from selling at $t = 1$ and $t = 2$. The bracketed part of the third term is the price at which the regular investor expects to be able to sell IPO shares at $t = 2$.

Assuming, as before, that the issuer does not need to raise a particular level of financing, the objective remains to maximize the excess value, Π , of offered shares over their ‘true’ (long-term) value, subject to the participation constraint defined in (4). Thus, the issuer solves

$$\begin{aligned} \max_{P_0, Q} \Pi &\equiv Q (P_0 - V_R) \\ \text{s.t.} \quad &-QP_0 + q_1^*P_1 + q_2^*[(1 - \gamma)E^s(P_2) + \gamma V_R] \geq 0 \end{aligned}$$

Lemma 1 *The participation constraint will always be binding.*

Proof. Suppose not. That is, the optimal P_0 and Q are such that the constraint has slack. Then the issuer can increase P_0 and so increase his profits, which contradicts the optimality of P_0 and Q . ■

Using the lemma the issuer’s objective function simplifies to

$$\max_Q \Pi \equiv [q_1^*P_1 + q_2^*E^s(P_2)(1 - \gamma) + q_2^*V_R\gamma] - QV_R$$

where q_1^* and q_2^* are given by (3). The first bracketed term on the right-hand side is the maximum amount that a regular investor is willing to pay for the IPO shares, from the participation constraint in (4).

From Proposition 1, we know $q_1^* \leq \bar{Q}_1$. Thus, P_1 is determined by s -type investors, on the basis of their expectation regarding P_2 . Using $P_1 = E^s(P_2) = V_R + a - \lambda Q$ in the above expression and simplifying, the issuer’s objective function can be written as

$$\max_Q \Pi \equiv [q_1^*(Q) + (1 - \gamma)q_2^*(Q)][a - \lambda(q_1^*(Q) + q_2^*(Q))]$$

where we explicitly recognize the dependence of q_1^* and q_2^* on Q .

We can now derive the issuer’s optimal offer size.

Proposition 2 *With a single regular investor, the issuer’s optimal choice of quantity Q^* to be issued is given by*

$$Q^* = \begin{cases} \frac{a}{2\lambda} - \frac{\bar{Q}_1\gamma}{2(1-\gamma)} & \text{if } \bar{Q}_1 < \frac{a(1-\gamma)}{\lambda(2-\gamma)} \\ \frac{a}{2\lambda} & \text{otherwise} \end{cases}$$

The resulting choices of q_1^* and q_2^* are such that

$$(q_1^*, q_2^*) = \begin{cases} \left(\bar{Q}_1, \frac{a}{2\lambda} - \bar{Q}_1 \left(1 + \frac{\gamma}{2(1-\gamma)} \right) \right) & \text{if } \bar{Q}_1 < \frac{a(1-\gamma)}{\lambda(2-\gamma)} \\ \left(\frac{a}{2\lambda}, 0 \right) & \text{otherwise} \end{cases}$$

Proof. We obtain the above expressions from first-order conditions obtained by taking the derivative of the firm's objective function with respect to q_2 . It can be shown that there is a unique maximum because the second order condition with respect to q_2 is negative. ■

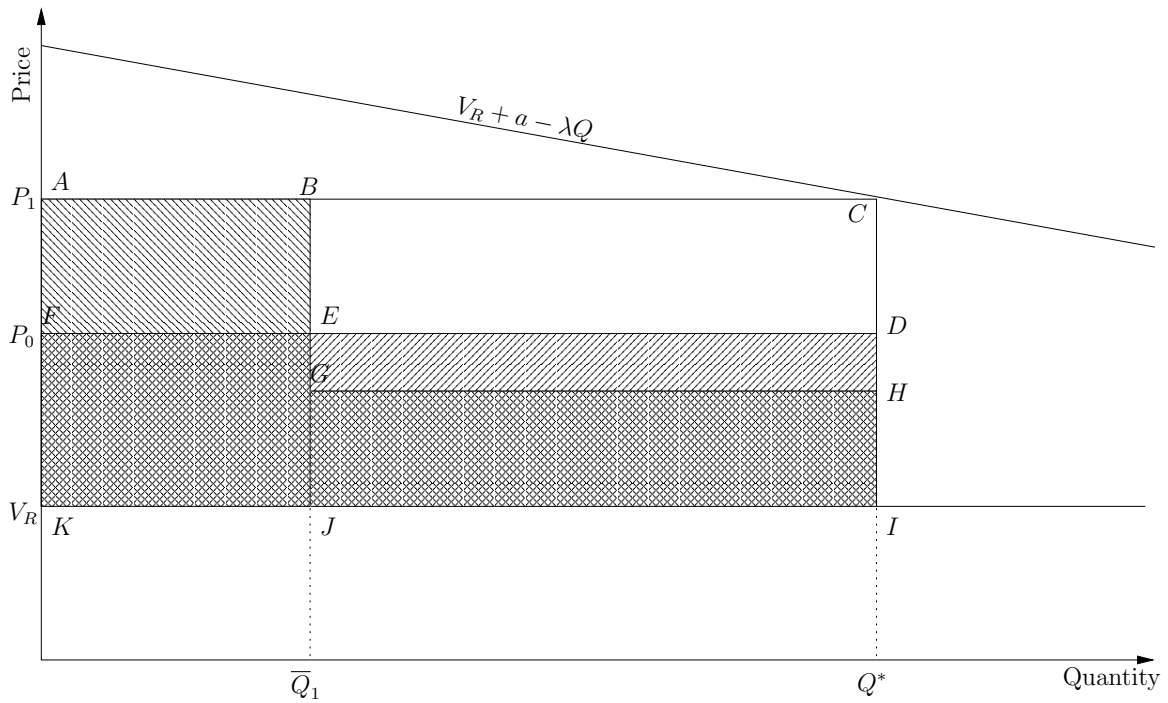
We now turn to pricing. The issuer needs the regular investor to hold inventory if \bar{Q}_1 is small (relative to total demand by sentiment investors), *i.e.* less than $\frac{a(1-\gamma)}{\lambda(2-\gamma)}$. So long as the hot market persists, the regular investor sells his inventory to newly-arriving sentiment investors at $t = 2$. If the hot market ends, he is left with shares priced at V_R . For a regular investor to accept this negative-valued gamble, the initial offer price needs to be set at a discount relative to the price at which the shares are expected to trade initially, so that $P_0 < E^s(P_2) = P_1$. In our model, the share price will eventually drift to V_R , where $V_R < P_0$ from the binding participation constraint of the investor. Thus, with a regular investor holding inventory that he disposes of over time, *both* an initial price run-up (underpricing) and long-run underperformance will be observed. These patterns can be viewed as arrangements that have, in effect, evolved as a means to maximize value extraction from *s*-type investors.

If \bar{Q}_1 is large (relative to total sentiment investor demand), there are no benefits from having a regular investor hold inventory and the offering being underpriced. Thus, the presence of sentiment investors is a necessary but not sufficient condition for the initial price run-up. The long-run return $(V_R - P_1)/P_1$, on the other hand, is always negative in our set-up. It results from the overly optimistic valuation of sentiment investors and represents market inefficiency – sustained by the difficulty and cost of establishing short positions in the stock. By implication, we do not expect a monotonic relation between underpricing and the long-run price drift.

Proposition 3 summarizes the above discussion regarding the predicted price patterns. Figure 1 illustrates.

Figure 1: Issuer Surplus

The figure illustrates two different selling mechanisms when the optimal quantity chosen (Q^*) is strictly greater than the sentiment demand at $t = 1$ (\bar{Q}_1). First, suppose the issuer can sell in stages directly to the investors (as modeled in Section 3). The s-type investors present at $t = 1$ rationally anticipate demand at $t = 2$ and price the security at P_1 . At $t = 2$, the hot market persists with probability $(1 - \gamma)$, in which case the issuer sells quantity $(Q^* - \bar{Q}_1)$ at price $P_2 = P_1$. If the hot market ends, he is forced to sell the shares at their fundamental value V_R . The rectangle $GHIJ$ represents the expected surplus obtained at $t = 2$, which is equal to $(1 - \gamma)(P_1 - V_R)(Q^* - \bar{Q}_1)$. The issuer's total surplus is given by the area in the two rectangles $ABJK$ and $GHIJ$. Second, if the issuer is prevented from directly selling in stages, but he can obtain the cooperation of a regular investor, we have the case modeled in Section 4. The issuer sells Q^* shares to the regular investor at price P_0 . At $t = 1$, the regular investor obtains a profit equal to the rectangle $ABEF$ and at $t = 2$ suffers an expected loss equal to the rectangle $DEGH$. The zero profit condition on the investor ensures that the gain at $t = 1$ is equal to the expected loss at $t = 2$, leaving the issuer with the same profits as in the earlier case. Underpricing arises because the regular investor needs to be compensated for the expected inventory loss, and so $P_0 < P_1$. Long-term underperformance arises because the issuer always extracts some surplus from the sentiment investors, and so $P_0 > V_R$.



Proposition 3 *With a single regular investor,*

1. *if \bar{Q}_1 is small enough such that $q_2^* > 0$, then the IPO shares will exhibit an initial price run-up: $P_0 < P_1$;*
2. *if \bar{Q}_1 is large, then the shares will not exhibit an initial price run-up: $P_0 = P_1$;*
3. *$\forall \bar{Q}_1$ the long-run return will be negative: $V_R < P_1$.*

Though optimistic on the issuer's stock, sentiment investors, in our model, are still acting rationally given their beliefs: they correctly anticipate the arrival of more sentiment investors at $t = 2$ (albeit with the wrong probability) and price the stock accordingly. If the sentiment investors were not forward-looking in this sense, then the price at $t = 1$ would be determined by the marginal sentiment investor present at $t = 1$. In that case, the price run-up would, in fact, be even higher than that predicted by our existing set-up.

We can make a more precise prediction regarding the relative magnitudes of underpricing and long-run performance:

Proposition 4 *With a single regular investor, the initial price run-up $[P_1 - P_0]$ and long-run price drift $[P_1 - V_R]$ will be related as follows:*

$$P_1 - P_0 = \frac{\gamma q_2}{Q}(P_1 - V_R).$$

Proof. See the appendix. ■

For expositional ease, we will refer to the ratio of the initial price run-up and the long-run price drift as the 'price reversal ratio'. From Proposition 4, the price reversal ratio is proportional to the inventory carried by the regular investor as a fraction of offer size. Thus,

$$\text{Price Reversal Ratio} \equiv \frac{P_1 - P_0}{P_1 - V_R} = \gamma \frac{q_2}{Q}.$$

4.2 Comparative Statics

We now study the properties of the first-day return, long-run performance, and the price reversal ratio. We focus on two parameters of interest: the intercept of the sentiment investors'

demand function (a) and the probability of the hot market coming to an end (γ). In the context of the model, both parameters are exogenous and affect the nature of the hot market.

Proposition 5 *With a single regular investor,*

1. *the number of shares issued, the first-day return, and the price reversal ratio are all increasing, while long-run performance is decreasing, in the sentiment (a) of the market;*
2. *long-run performance and the number of shares issued is decreasing in γ ; and*
3. *the first-day return and the price reversal ratio are increasing in γ for low γ .*

Proof. See the appendix. ■

An increase in the intercept of the demand function, a , can be interpreted as an increase in the optimism of the sentiment investors. As one might expect, issuers in our model respond by increasing the size of the offering. The prediction on the first-day return, however, is not obvious. It may seem anomalous that a more bullish market does not translate into a *smaller* first-day return: why don't issuers take advantage of the bullishness of the market and increase the offer price, resulting in a smaller first-day return? The reason why the first-day return increases with market sentiment is that underpricing is a way of compensating the regular investor for taking on the risk of the hot market crashing at $t = 2$. As offer size increases, the fraction of the offering carried over to $t = 2$ also increases. Consequently, the regular investor needs to be compensated more (on a per share basis) for taking on the risk of carrying this inventory.

An increase in γ , the probability of market sentiment turning sour, reduces the expected gain from holding inventory until $t = 2$. As a consequence, the issuer is better off reducing the quantity of shares issued. However, a reduction in the quantity issued increases the price at $t = 1$, thus worsening long-run performance.

An increase in γ has two opposing effects on the first-day return. First, it increases the regular investor's required compensation due to the direct effect of an increase in the probability of a crash. Second, the indirect effect of a reduction in the quantity issued is to reduce the

inventory the regular investor holds. Proposition 5 shows that the first effect dominates for low γ as the percentage change in q_2^* for low γ is small. For high enough γ , q_2^* goes to zero and so the first-day return disappears. For intermediate levels of γ , the change in the first-day return is ambiguous. Similar characteristics are inherited by the price reversal ratio.

4.3 Discussion

In this section, we have developed an alternative to the direct-sale mechanism described in Section 3. Our alternative mechanism requires the regular investor to carry inventory for sale in the secondary market. It is important to understand that both mechanisms give the issuer exactly the same expected proceeds, even though the delegated-inventory mechanism involves underpricing. This simply follows from the zero-profit condition in Lemma 1. In words, in the delegated-inventory mechanism, the issuer underprices the stock to compensate the regular investor for bearing the risk of the sentiment demand evaporating too soon. Thus, underpricing is not a value transfer from the issuer to the regular investor; it is a fair payment for the regular's expected loss. In the direct-sale mechanism, the issuer bears the exact same risk himself. Noting that everyone is risk-neutral, it is straightforward to show that the expected proceeds from the two mechanisms are equivalent. Figure 1 illustrates.

In some sense, the direct-sale mechanism described in Section 3 resembles an IPO followed – if the sentiment demand survives – by an SEO. Couldn't the issuer improve on the delegated-inventory mechanism by conducting an SEO shortly after the IPO? The answer is no: the expected proceeds are at best the same (ignoring transaction costs for the SEO) or, more realistically, strictly lower (net of transaction costs). Leaving aside transaction costs, if sentiment demand develops over several periods (perhaps stirred by the buzz of the IPO), it is clearly impractical for the issuer to take advantage of it via a sequence of possibly small SEOs. The regular investor, on the other hand, faces no constraints on the frequency or size of after-market sales, and so can optimally take advantage of sentiment investors as and when they arrive in the market. Thus, while we do not rule out an SEO soon (within a few weeks) after the IPO, we argue the issuer can better take advantage of developing sentiment demand by obtaining the regular investor's cooperation than by planning to do multiple SEOs.

5 Extensions

We now outline two extensions to the model. In the previous section we analyzed a very tractable model to understand the properties of the initial price run-up when issuers optimally take advantage of the sequential arrival of sentiment investors. In Section 5.1, we generalize the model to show that similar results obtain if sentiment investors arrive over many periods. The extension highlights the impracticality of an issuer doing a series of equity offerings as demand evolves over multiple periods, compared to using the inventory holding mechanism.

In Section 5.2, we examine the strategy of underwriters who have to pay rents to induce cooperative behavior among multiple regular investors.

5.1 Multi-Period Sentiment Demand

We now extend the model to incorporate sentiment demand that arises over several periods, say weeks or months. The set-up captures the notion that as potential sentiment investors hear the buzz, some are likely to invest in the stock. The arrival of future sentiment investors, though likely, is still uncertain. This will be reflected in the setting of the offer price.

We assume that new sentiment investors may arrive every period after the IPO. The demand, however, decays over time at rate α . Specifically, we assume

$$Q_t = \alpha Q_{t-1}, \quad \alpha < 1.$$

As in the two-period model analyzed earlier, if the sentiment demand has survived up to period t then with probability γ it will disappear in that period. We maintain all other assumptions of the two-period model developed earlier. Thus, the marginal sentiment investor's reservation value is given by $V_R + a - \lambda Q$, sentiment investors account for the arrival of future sentiment investors, and they do not share the regular investor's belief about the possibility of the hot market ending. We assume that the number of shares issued is sufficient to satisfy sentiment demand for up to S periods. Note that we are characterizing the optimal quantity to be sold in terms of the number of periods. The reason we can do this is that, for a given quantity to be sold, the optimal selling strategy (as in Proposition 1) is to sell whatever can be absorbed

by sentiment investors each period till the hot market ends or else the allocation is fully sold.

Thus, the number of shares issued, Q , is given by

$$\begin{aligned} Q &= Q_1 + \alpha Q_1 + \alpha^2 Q_1 + \dots + \alpha^{S-1} Q_1 \\ &= \frac{1 - \alpha^S}{1 - \alpha} Q_1 \end{aligned}$$

A single regular investor, who is allocated Q shares at a price P_0 , sells Q_1 shares at $t = 1$, if the hot market persists αQ_1 shares at $t = 2$, and so on. The break-even condition implies

$$\begin{aligned} (P_0 - V_R) Q &= Q_1 (P_1 - V_R) + (1 - \gamma) \alpha Q_1 (P_1 - V_R) + \dots + (1 - \gamma)^{S-1} \alpha^{S-1} Q_1 (P_1 - V_R) \\ &= \frac{1 - (1 - \gamma)^S \alpha^S}{1 - (1 - \gamma) \alpha} Q_1 (P_1 - V_R) \\ &= \frac{1 - (1 - \gamma)^S \alpha^S}{1 - (1 - \gamma) \alpha} Q_1 (a - \lambda Q) \end{aligned}$$

Substituting for Q , we obtain

$$(P_0 - V_R) Q = \frac{1 - (1 - \gamma)^S \alpha^S}{1 - (1 - \gamma) \alpha} \left(a - \lambda \frac{1 - \alpha^S}{1 - \alpha} Q_1 \right) Q_1.$$

Thus, the issuer's problem is to solve

$$\max_S \frac{1 - (1 - \gamma)^S \alpha^S}{1 - (1 - \gamma) \alpha} Q_1 \left(a - \lambda \frac{1 - \alpha^S}{1 - \alpha} Q_1 \right).$$

We denote the optimal S by S^* . At the optimum,

$$\begin{aligned} Q^* &\equiv Q(S^*) = \frac{1 - \alpha^{S^*}}{1 - \alpha} Q_1 \\ P_0 &= V_R + \frac{1 - (1 - \gamma)^{S^*} \alpha^{S^*}}{1 - (1 - \gamma) \alpha} \left(\frac{1 - \alpha}{1 - \alpha^{S^*}} \right) (a - \lambda Q^*) \\ P_1 &= V_R + a - \lambda Q^* \end{aligned}$$

From these expressions it is easy to see that our results on the existence of an initial price run-up and long-run underperformance will go through in a multiple-period setting. The following proposition summarizes without proof.

Proposition 6 *If the sentiment demand evolves over multiple periods and the underwriter has access to a single regular investor, then*

1. if Q_1 is sufficiently small such that $S^* > 1$, then the IPO shares will exhibit an initial price run-up: $P_0 < P_1$;
2. $\forall Q_1$ the long-run return will be negative: $V_R < P_1$.

Obtaining the comparative statics in closed-form is not feasible in general. Given that the multi-period model is not as tractable as the two-period model analyzed earlier, we resort to providing numerical solutions for selected parameter values. We solve the problem for the following parameter values: the long-term value V_R is 5; the probability of the hot market ending in any period, γ , is 10%, which is roughly equivalent to a 10-period expected length of the hot market; sentiment demand is assumed to decay at rate $\alpha = 10\%$; the initial demand Q_1 is normalized to 1 unit; and the slope of the demand curve λ is 0.5. Given the above parameter values we numerically solve for the optimal S and plot the predicted price patterns as a function of the level of optimism (a) in Figure 2.

Figure 2 shows that the first-day return and the price reversal ratio are both increasing in a . Long-run performance is always negative and decreasing in the level of optimism. The intuition is similar to the one provided earlier. An increase in optimism among sentiment investors makes it optimal for the issuer to increase issue size, which implies that the regular investor has to carry more inventory and bear a greater expected loss if the hot market ends.

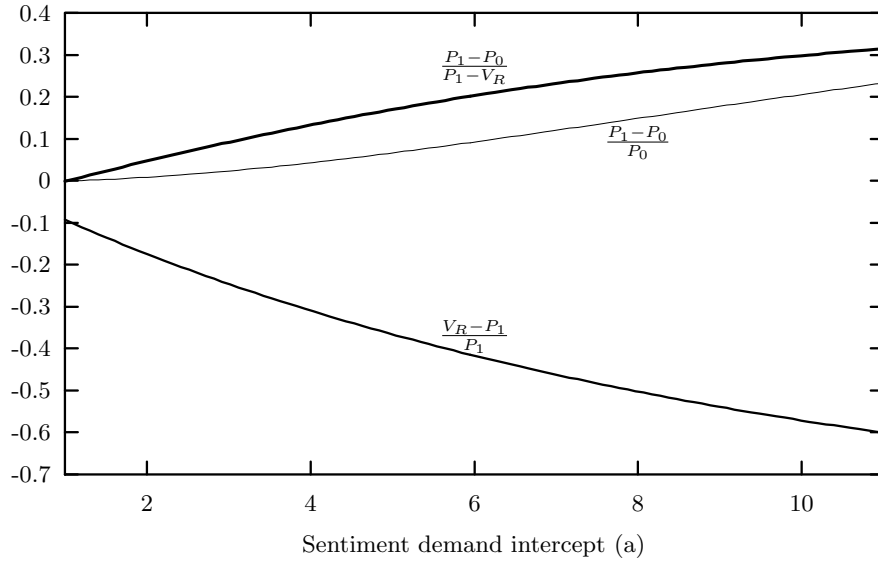
Figure 2 suggests, therefore, that the qualitative nature of our results is unaffected by an extension to many periods. We also believe that the multiple period extension better captures the notion that the disposal of share allocation by the regular investor is gradual and takes place over a number of periods – making it less plausible that an alternative procedure requiring the issuer to do multiple SEOs would be similar or more efficient.

5.2 Limited Ability to Obtain Cooperation from Institutional Investors

We have so far considered the case of a monopolist regular investor. Being a monopolist, the investor has an incentive to cooperate with the underwriter, by holding inventory and delaying the sale of part of his IPO allocation. However, if there are many regular investors, say N , they face a free-rider problem. Collectively, regular investors are better off holding on to their

Figure 2: Plot of first-day return, long-term performance and price reversal ratio

In this figure we plot the first-day return $[\frac{P_1 - P_0}{P_0}]$, the long-term performance $[\frac{V_R - P_1}{P_1}]$, and the price reversal ratio $[\frac{P_1 - P_0}{P_1 - V_R}]$. We solve for the optimal S and calculate P_1 and P_0 at the optimal S for the following parameter values: the long term value (V_R) is 5; the probability of the hot market ending in any period, γ , is 10%, which is equivalent to a 10-period expected length of the hot market; the demand is assumed to decay at a rate (α) of 10%; the initial demand (\bar{Q}_1) is normalized to 1 unit; and the slope of the demand curve (λ) is 0.5.



inventory until $t = 2$. However, individually each can benefit by unloading his entire allocation at $t = 1$. Hence, an underwriter's ability to induce cooperative behavior is determined by the extent to which he can offer inducements or threaten punishment. A likely form of punishment is the threat of exclusion of regular investors from future IPOs (or other desirable deals). Such an exclusion will impose a cost on the regular investors only if they obtain non-zero rents from IPO allocations. Given the clamor to obtain IPO allocations witnessed in the late 1990s, it seems reasonable that regular investors do obtain rents. In this section, we generalize the model to explicitly allow for such rents.

We assume the underwriter can extract some rents on behalf of his regular investors. These rents can be viewed as the outcome of a bargaining game between the issuer and the underwriter and in general would depend on the level of competition in the IPO market. We denote the per share rent by r . Given these rents, an underwriter can impose penalties on regular investors by excluding them from future allocations of IPO shares – thereby deterring deviation from the inventory holding strategy. The extent of punishment depends on the magnitude of r and the anticipated frequency of future IPO allocations. Specifically, we assume regular investors' valuation of such future benefits is $r\pi$, where π is the multiple that accounts for the probability and timing of future IPOs. One would expect an investment bank with a bigger market share to have a higher π . Similarly, if the market believes the hot market to continue for some time, one would expect π to be high. Conversely, if the near-term outlook for the IPO market is bleak, or if the underwriter's market share is small, exclusion from future IPOs will provide only limited incentives for inventory holding.

Let \hat{P}_0 be the offer price that incorporates the rent r . Thus,

$$\hat{P}_0 = P_0 - r$$

where P_0 , as defined in Section 3, is the offer price for $r = 0$.

On the margin, regulars can choose to sell a share at price P_1 at $t = 1$, or sell at $t = 2$ and expect to get $E^R(P_2) = \gamma V_R + (1 - \gamma)E^S(P_2)$. The potential loss from future exclusion from the IPO process, $r\pi$, must be large enough to deter deviation from the inventory holding

strategy. Therefore, we need

$$r\pi \geq \frac{q_2}{N} (P_1 - E^R(P_2))$$

where $\frac{q_2}{N}$ represents the inventory each investor carries to $t = 2$. Denoting $R \equiv r\pi N$ we can express the above constraint as

$$R \geq q_2 (P_1 - \gamma V_R - (1 - \gamma)E^s(P_2)). \quad (5)$$

Substituting for P_1 from (2) in (5) the constraint reduces to

$$R \geq \gamma q_2 (a - \lambda(q_1 + q_2)). \quad (6)$$

The analysis presented in Section 4 corresponds, therefore, to the case where the above constraint has slack. We now consider the situation in which the constraint is binding, *i.e.* in which (6) is violated at the optimal q_1^* and q_2^* . The next proposition shows that the constraint is more likely to be violated when market sentiment is more exuberant or when the market has a higher probability of crashing.

Proposition 7 *The expected gain to regular investors of deviating from the inventory holding strategy and selling shares at $t = 1$,*

$$\gamma q_2^*(a, \gamma) [a - \lambda(q_1^*(a, \gamma) + q_2^*(a, \gamma))]$$

is increasing in a and γ . Thus, if constraint (6) is violated at (q_1^, q_2^*) for some $a = \hat{a}$ and $\gamma = \hat{\gamma}$, then it will be violated for all $a > \hat{a}$ and $\gamma > \hat{\gamma}$.*

Proof. See the appendix. ■

The gain from deviating from the inventory holding strategy depends on the product of q_2 and $(P_1 - V_R)$. An increase in a increases both (Proposition 5), increasing the incentive to deviate as indicated in Proposition 7. Similarly, an increase in the probability of a crash γ increases the incentive of regular investors to deviate by selling their entire allocation of IPO shares at $t = 1$.

The issuer's constrained problem is to solve the following:

$$\begin{aligned} & \max_{q_1, q_2} (q_1 + (1 - \gamma) q_2) (a - \lambda(q_1 + q_2)) \\ & \text{s.t. } R \geq \gamma q_2 (a - \lambda(q_1 + q_2)). \end{aligned}$$

Let the solution to the above programming problem be (q_1^c, q_2^c) . The next proposition characterizes the quantities chosen by the issuer.

Proposition 8 *If the optimal (q_1^*, q_2^*) defined in Proposition 2 are such that (6) is violated, then the optimal choice of shares issued (q_1^c, q_2^c) is given by*

$$\begin{aligned} q_1^c &= \bar{Q}_1 \\ q_2^c &= \frac{1}{2\lambda} \left((a - q_1\lambda) - \sqrt{(a - q_1\lambda)^2 - \frac{4R\lambda}{\gamma}} \right). \end{aligned} \quad (7)$$

Proof. See the appendix. ■

The optimal quantity sold in the secondary market at $t = 1$ is the same as that in the earlier unconstrained case. This is because if more than \bar{Q}_1 were sold at $t = 1$, the marginal investor would no longer be a sentiment investor but instead a rational investor. However, constraint (6) does decrease the quantity sold at $t = 2$, and consequently the total issue size. This distortion in q_2 is highest for underwriters with a small R : with a smaller amount of potential rent at stake, incentive compatibility requires regular investors to carry fewer IPO shares in inventory. Thus, banks with small R have less IPO placing capacity and so do smaller deals.

The positive relation between R and q_2 in equation (7) has one further implication. If periods of high IPO volume imply increases in R , the size of the IPOs will also be larger, ceteris paribus. Similarly, underwriters who gain (or are expected to gain) larger market shares can impose bigger penalties, *i.e.*, they have a higher R . All else equal, this allows them to increase the size of their offerings. Thus, growth will beget more growth and a hot market will get hotter. This suggests that a hot market can have a certain self-fulfilling logic.

In the next proposition we analyze the impact of R on the IPO price patterns when the inventory holding constraint is binding.

Proposition 9 *If the number of shares issued Q is such that regular investors' inventory holding constraint is binding, then the first-day return $(P_1 - \hat{P}_0)/\hat{P}_0$, long-run performance $(P_1 - V_R)/P_1$, and the price reversal ratio $(P_1 - \hat{P}_0)/(P_1 - V_R)$ are all increasing in R .*

Proof. See the appendix. ■

The positive relation between the first-day return and R predicted in Proposition 9 may seem surprising, for it implies that IPOs lead-managed by more active or more prestigious underwriters are more underpriced.¹³ Recall that underpricing serves as a form of compensation to the regulars for carrying inventory. An underwriter with a lower R can induce only a relatively small amount of inventory holding q_2^c , as shown in Proposition 8. The less inventory is carried, the less need there is for the offering to be underpriced.

That lower R offerings are associated with worse long-run performance is immediate, since the decrease in q_2 (and thus in Q) increases the $P_1 = V_R + a - \lambda Q$ that sentiment investors are willing to pay. This prediction is generally consistent with the empirical evidence that IPOs done by larger, more established underwriters tend to exhibit better long-term performance.

6 Empirical Implications

Our model has a number of empirical implications, some of which have already been mentioned. We now collect these and other empirical implications. Several of them are consistent with existing empirical evidence, while others are novel and untested.

Prediction 1 (Long-run performance) *Firms taken public in a hot market subsequently underperform, both relative to the first day trading price P_1 and the offer price P_0 .*

Underperformance relative to P_1 is not a surprising prediction; it follows from the twin assumptions of sentiment investors and limits to arbitrage. Underperformance relative to P_0 is a stronger claim. It follows because the offer price will exceed fundamental value V_R by an amount equal to the issuer's share in the surplus extracted from the sentiment investors. Purnanandam and Swaminathan (2001) lend support to our prediction that the offer price can exceed fundamental value. They show that compared to its industry peers' multiples, the median IPO firm in 1980-1997 was overpriced at the offer by 50%. Interestingly, it is the firms that are most overpriced in this sense which subsequently underperform.

¹³As we will discuss later, recent empirical evidence tends to support this prediction. However, certification arguments imply the opposite relation.

In a cold market, there are no exuberant investors and so prices are set by rational investors at fundamental value. Thus, in our model, there is neither underpricing nor long-run underperformance in a cold market. The empirical evidence is consistent with this concurrence of hot markets and poor long-run performance. Ritter (1991) shows that companies that went public in the hot market of the early 1980s experienced very high underpricing and performed particularly badly in the long-run, a result corroborated more generally by Loughran and Ritter (1995) and Helwege and Liang (1996). Furthermore, Loughran and Ritter (2000) find strong evidence that companies going public in high-volume periods experienced significantly worse long-run performance than those floated in low-volume periods. They view this finding as evidence of issuers taking advantage of periods of misvaluation.

Taken at face value, Prediction 1 implies that all companies floated during a hot market will underperform. Of course, a hot market could be confined to a particular industry or industries. Thus, there is no reason to believe that *all* companies floated at the same point in time will necessarily underperform.

To the extent that limits to arbitrage are more severe among small-cap stocks, we would expect long-run underperformance to be concentrated among small IPOs and largely absent among larger ones. The findings of Brav and Gompers (1997) and others are consistent with this prediction.

Prediction 2 (Partial adjustment) *As the difference in opinion between rational and sentiment investors increases, both the offer price and underpricing increase.*

This follows directly from Proposition 5. An increase in a , reflecting an increase in the optimism of sentiment investors, results in an increase in the offer price P_0 and in the initial price run-up. Prediction 2 implies a positive correlation between pre-market changes in the offer price and after-market underpricing. This is consistent with the empirical evidence presented in Hanley (1993) who shows that underpricing is higher, the more the offer price exceeds the midpoint of the original indicative price range. This ‘partial adjustment’ phenomenon is often viewed as supporting the information revelation model of Benveniste and Spindt (1989): to induce truthful revelation, underwriters must leave more money on the table in states of the

world where investors hold particularly positive information. Our model provides an alternative rationale for the partial adjustment phenomenon based on an increase in the difference in opinion between sentiment and rational investors after the original price range is set. Since our model does not rely on private information, it can accommodate Loughran and Ritter’s (2002) finding of partial adjustment to *public* information.

To see what is driving partial adjustment – increases in investor optimism or information revelation – requires a measure of the degree of divergence of opinion in the IPO market. Aggarwal and Conroy (2000) propose time-to-first-trade as a proxy: delaying the first trade may enable the underwriter to better gauge market demand and could thus be an indication of greater initial divergence of opinion. They document that underpricing increases in time-to-first-trade, consistent with our prediction that underpricing increases in the degree of divergence of opinion in the IPO market. However, whether this is behind the partial adjustment phenomenon remains to be investigated.

Prediction 3 *As the difference in opinion between rational and sentiment investors increases, long-run performance worsens.*

This follows from Propositions 5 and 9. Like Prediction 2, this prediction requires a measure of divergence of opinion. Using Aggarwal and Conroy’s time-to-first-trade proxy, Houge, Loughran, Suchanek, and Yan (2001) show that late-opening IPOs significantly underperform over the subsequent three years. Houge et al. further analyze the relation between long-run performance and the size of the bid-ask spread set by market-makers when the IPO opens. The bid-ask spread should at least in part reflect the market-maker’s uncertainty regarding a company’s value. Consistent with this interpretation, Houge et al. find that subsequent abnormal performance decreases in the opening spread. Dunbar (1998) finds that IPOs with positive price and offer size adjustments are prone to poor long-run performance and conjectures that this is evidence of “excess initial retail investor demand.”

Rajan and Servaes (1997) look at analyst following after the IPO and find that not only were analysts over-optimistic about earnings and long-term growth prospects, but issuers may also have taken advantage of windows of opportunity: more companies went public when analysts

where particularly over-confident about recent IPOs in the same industry. Interestingly, IPOs with low forecast growth rates subsequently out-performed IPOs with high forecast growth rates, by a margin of more than 100% over five years. To the extent the forecasts reflected some of the optimism of the sentiment investors, these findings are consistent with our prediction.

In our model, as γ , the likelihood of the hot market ending, increases, regular investors hold less inventory, indicating that a relatively larger fraction of the allocation is flipped. This is consistent with the results in Krigman, Shaw, and Womack (1999). Using the ratio of sell-initiated large-block trading volume to total volume traded on the first day as a measure of the sentiment of institutions, they find that IPOs which are flipped more on the first day underperform low-flipping IPOs over the next twelve months.

Prediction 4 *The relation between long-run performance and the initial price run-up is non-monotonic. It is negative if the probability of the hot market ending is small.*

This follows from Propositions 5 and 9. Prediction 4 may explain the relatively mixed extant evidence on this point. Ritter (1991) finds weak evidence that underpricing and long-run performance are negatively correlated. In particular, he shows that long-run performance is particularly poor amongst smaller issuers, which tend to have the highest initial returns. Focusing on the recent boom in internet IPOs, Ofek and Richardson (2002) find a strong negative relation between first-day returns and future excess returns to the end of 2000. Krigman, Shaw, and Womack (1999), on the other hand, find a positive relation between underpricing and one-year returns, except for ‘extra-hot’ IPOs: offerings with initial returns in excess of 60% have the worst one-year performance in their sample.

The dynamics of the IPO market cycle

Prediction 5 *As the optimism of sentiment investors increases, more companies have an incentive to go public (to take advantage of the optimistic investors) and offer sizes increase.*

Lee, Shleifer, and Thaler (1991) show that the annual number of IPOs between 1966 and 1985 was strongly negatively related to the discount on closed-end mutual funds, which they argue is a measure of the sentiment of retail investors. Similarly, Lowry and Schwert (2002) show that following periods of ‘unusually’ high underpricing, both IPO volume and IPO reg-

istrations increase and that companies which are already in SEC registration accelerate the completion of their IPOs. This is consistent with the first part of Prediction 5. We know of no existing evidence regarding changes in offer size over the IPO market cycle, although Ljungqvist and Wilhelm (2003) do document increases in mean and median issue size among U.S. IPOs in every year as the IPO market became hotter between 1996 and 2000.

We also conjecture that as the IPO market heats up, lower-quality companies may go public for opportunistic reasons, resulting in a decline in the quality of the average issuer. The hot market of 1998-2000 may be a good illustration of the evolution of issuer quality over the IPO market cycle. According to Ljungqvist and Wilhelm (2003), 61.6% of firms listing in the U.S. in 1997 had 12-month track records of earnings; by 1999 this had fallen to just 23.6%. Helwege and Liang (1996) specifically examine the quality of IPO firms in ‘hot’ and ‘cold’ markets. Interestingly, and contrary to our conjecture, they find no difference in operating performance (their measure of issuer quality) between hot-market and cold-market issuers.

The role of the underwriter

Prediction 6 *More prestigious underwriters have access to higher IPO deal flow and so have higher R . Higher R , in turn, leads to larger initial price run-ups and better long-run performance.*

The evidence on underpricing is mixed. Contrary to our prediction, Carter and Manaster (1990) and Carter, Dark, and Singh (1998) find that more prestigious underwriters are associated with lower underpricing. Beatty and Welch (1996), on the other hand, point out that this relation appears to be reversed in the 1990s. Habib and Ljungqvist (2001) show that the apparent reversal is driven, at least in part, by the failure to treat the choice of underwriter as endogenous. Prediction 7 applies in particular to hot markets whereas none of the above papers control for the state of the IPO market. Benveniste, Ljungqvist, Wilhelm, and Yu (2003) find a positive relation between underpricing and underwriter prestige in the 1999/2000 hot market, consistent with our prediction.

Furthermore, Carter, Dark, and Singh (1998) show that IPOs lead-managed by more prestigious underwriters are associated with lower underperformance over the next three years.

Supporting the equilibrium

Prediction 7 (Allocation policy) *Underwriters have a preference for selling to regular (typically institutional) investors.*

This prediction follows because the repeated interaction with regular investors, and the ease of tracking larger positions, will lower the costs of sustaining the equilibrium. Empirical evidence suggests that IPO allocations are heavily skewed in favor of institutional investors [Hanley and Wilhelm (1995)] and that regular investors are favored over infrequent investors [Cornelli and Goldreich (2001)].

The extension to a setting of multiple regular investors considered in Section 5.2 suggests that underwriters also have a preference for targeting a select, probably small group of institutions. All else equal, and for a given amount of rent, a smaller group makes it easier to obtain the institutions' cooperation. Targeting a narrow subset of investors is a common feature of U.S.-style bookbuilding.

Prediction 8 (Flipping) *Underwriters penalize investors who engage in excessive flipping (relative to the optimal selling strategy).*

The prediction is consistent with the use of penalty bids [Aggarwal (2000)] which underwriters impose on syndicate members whose clients flip their allocations. Subtler penalties include exclusion from future IPO offerings. Such penalties are usually viewed as part and parcel of price support. Our model predicts that the penalties should occur more widely than in IPOs which receive price support. This remains to be tested.

Boehmer and Fishe (2001) report that institutional investors tell underwriters in advance whether or not they intend to flip, which is consistent with underwriters tolerating some amount of flipping among institutions, rather than penalizing all flipping indiscriminately.

Prediction 9 (Flipping) *Penalties for excessive flipping are targeted more heavily at retail and infrequent investors.*

In order to sustain the equilibrium, underwriters need to ensure that their regular investors do not make (excessive) losses on their holdings between dates 1 and 2. Competing selling pres-

sure from investors who are not party to the equilibrium strategy would therefore undermine the equilibrium. Articles in the business press provide some anecdotal support for Prediction 9: “When a stock quadruples on its first day of trading, many stockholders want to sell, or flip, their shares [...]. They can, if they are big investors or mutual funds that have close ties with the underwriter. [...] But underwriters force most individual investors – and even money managers without much clout – to hold on to their shares for as long as 90 days.”¹⁴

Such practices have attracted the attention of securities regulators. In August 1998, the SEC “launched an inquiry into how brokerage firms seek to bar small investors from flipping [...] initial public offerings, but allow institutional investors to flip hot new stocks with impunity.”¹⁵ At the same time, Massachusetts regulators fined Joseph Charles and Associates, a Florida-based underwriter, \$50,000 and barred it from doing business in the state for two years, because of the bank’s policy against flipping by retail investors.

Prediction 10 (Post-IPO sales) *Over time institutional investors unload their excess inventory. Hence, we predict a gradual divestment of IPO shares held by institutions and an increase in the shares held by retail investors.*

Boehmer and Fishe (2001) find that more than 92% of all first-day flipping transactions by investors who were allocated stock in the IPO are smaller than 10,000 shares. This strongly suggests that the buyers in these transactions are retail investors. There is more flipping in more underpriced offerings, consistent with our model. Krigman, Shaw, and Womack (1999) show that large (presumably institutional) investors are more active flippers (consistent with Prediction 9), and that they flip IPOs that perform the worst in the future. Field (1995) shows that long-run performance is better, the larger institutional stockholdings at the end of the first quarter of listing. Field does not have data on allocations, but her evidence is consistent with the prediction that institutions quickly sell out of the more marginal IPOs, so that by quarter’s end they hold more stock in the higher-quality companies. Dorn’s (2002) German data provide direct evidence in support of Prediction 10, by showing that the kinds of IPOs that retail investors overpay for the most, the hot IPOs, subsequently pass from institutional

¹⁴E. Leefeldt, “Fixed rates, double standards”, Bloomberg, May 2000, pp. 36-40.

¹⁵M. Siconolfi, “SEC launches probe into IPO flipping - state regulators fire 1st volley in matter.” The Wall Street Journal, August 20, 1998.

investors to retail investors.

Prediction 11 (Lock-ups) *Insiders will be released early from their lock-up provisions, a) if after-market demand from sentiment investors is unexpectedly high, b) once regular investors have unloaded their excess inventory, or c) if the hot market has come to an end.*

Several recent papers have documented that share prices fall significantly upon the expiry of lock-up provisions [Field and Hanka (2001), Brav and Gompers (2000), Ofek and Richardson (2000)], but their purpose has not previously been modeled. In our setting, lock-ups may serve to reassure institutional investors of their ability to sell at high prices in subsequent periods if the hot market persists, without having insiders compete to satisfy sentiment investors' demand.

According to Brav and Gompers (2000), early release is common: 60% of the firms in their sample have insiders sell shares prior to the lock-up expiry. The determinants of early release remain to be investigated. Our model suggests that such a release may be more likely under the circumstances mentioned.

Prediction 12 (Price support) *If demand by sentiment investors turns out to be unexpectedly low in the early after-market, the underwriter may want to absorb the excess supply by providing price support. The main beneficiaries of price support are the underwriter's regular investors.*

Benveniste, Erdal, and Wilhelm (1998) find that it is overwhelmingly large (presumably institutional) traders who execute sell orders in stabilized offerings, rather than small (presumably retail) traders. They interpret this finding as indicating that price support is offered mainly for the benefit of institutional investors. This is consistent with our underlying assumption that underwriters and large investors collude to extract value from sentiment investors.

Unexpectedly low demand at $t = 1$ does not necessarily mean that total demand by sentiment investors will be low. Thus, price support may well be profitable when there are unexpectedly many late-arriving sentiment investors at $t = 2$. Ellis, Michaely, and O'Hara (2000) investigate the profits of underwriters who act as market makers for their IPOs. The somewhat surprising fact that market makers' inventory losses do not differ between hot and

cold deals might be due to profitable trades with late-arriving sentiment investors.

7 Conclusions

Our model of the IPO process links some of the main empirical IPO ‘anomalies’ – underpricing, hot issue markets, and long-run underperformance – and traces them to a common source: the presence of a class of irrationally exuberant investors. The existence of such investors, coupled with short sale restrictions, leads to long-run underperformance. More interestingly, we resolve the apparent paradox that underpricing and long-run underperformance can coexist: after all, it is not obvious a priori why issuers do not take advantage of exuberance by raising offer prices, thus eliminating underpricing.

We show that the optimal selling policy, from the issuer’s point of view, usually involves staggered sales. Such staggered sales can be implemented by allocating the IPO to cooperative regular investors who hold inventory in the after-market for resale. IPO underpricing compensates regulars for the losses expected from holding inventory, given the probability of the hot market ending prematurely. The model is shown to be consistent with much of the – at times seemingly contradictory – evidence on IPOs. It also generates new, testable predictions about the IPO process.

The model raises interesting issues for future research as well. Consider, for instance, the extension of the model to a situation with multiple issuers, competing for a fixed supply of sentiment investors. In a competitive environment issuers may be unable to expropriate rents from the sentiment investors. If investment banks have market power, however, they may restrain the number of firms going public and the quantity of shares offered, in order to maintain IPO prices. Because of the obvious temptation to cheat among the *banks*, we expect to see punishment strategies that enforce a collusive equilibrium. We speculate that the use of investment banking syndicates may play a role, with punishment strategies taking the form of excluding cheating banks from future underwriting syndicates.

While our model does not specifically address social welfare, the possible expropriation of sentiment investors does give rise to some policy issues. To the extent that such expropriation

subsidizes risk-taking by young firms, social welfare may be enhanced. The downside, of course, is that, as the market heats up, some firms may go public for opportunistic reasons, purely to extract surplus from sentiment investors. This may involve firms with negative NPV investment opportunities. After sentiment collapses, the IPO market may effectively be shut for all but the most 'blue-chip' issuers, and some positive NPV projects may go unfunded. The social consequences of exuberant investors and their possible expropriation is, therefore, an open question. Do the exuberant provide subsidy to the socially productive – or are they merely lunch for the avaricious?

Appendix

Proof of Proposition 1:

For a given $Q > \bar{Q}_1$, suppose $q_1^* > \bar{Q}_1$. Consider $\hat{q}_1 = \bar{Q}_1$ and adjusting q_2 to \hat{q}_2 such that $\hat{q}_1 + \hat{q}_2 = Q$. P_2 is unchanged, as it is a function of Q . However, from (2), $P_1(\hat{q}_1) > P_1(q_1^*)$. Thus, $\Pi(\hat{q}_1) > \Pi(q_1^*)$. Similarly, for a given $Q > \bar{Q}_1$, suppose $q_1^* < \bar{Q}_1$. Consider $\hat{q}_1 = \bar{Q}_1$ and adjusting q_2 to \hat{q}_2 such that $\hat{q}_1 + \hat{q}_2 = Q$. $E(P_2)$ is unchanged, as it is a function of Q . In this case $P_1 = E^s(P_2)$, which is greater than $E^R(P_2)$. Thus, expected Π increases by $(E^s(P_2) - E(P_2))(\bar{Q}_1 - q_1^*)$. Hence, $q_1^* \neq \bar{Q}_1$ cannot be optimal. If $Q \leq \bar{Q}_1$, the non-negativity for q_2 implies $q_1^* = Q$ and $q_2^* = 0$. ■

Proof of Proposition 4:

From equation (4), the highest price P_0 a regular investor is willing to pay is

$$P_0 = P_1 \frac{q_1}{Q} + \frac{q_2}{Q} (P_1(1 - \gamma) + V_R \gamma)$$

Substituting this in $P_1 - P_0$, we obtain

$$\begin{aligned} P_1 - P_0 &= P_1 - \left[P_1 \frac{q_1}{Q} + \frac{q_2}{Q} (P_1(1 - \gamma) + V_R \gamma) \right] \\ &= P_1 \left[1 - \frac{q_1}{Q} - \frac{q_2}{Q} (1 - \gamma) \right] - V_R \gamma \frac{q_2}{Q} \\ &= \frac{\gamma q_2}{Q} (P_1 - V_R) \end{aligned}$$

Now, $P_1 = E^s(P_2) > V_R$. Thus, $(P_1 - V_R) > 0$. Therefore $(P_1 - P_0) > 0$ if and only if $q_2 > 0$ (or if and only if \bar{Q}_1 is small enough). ■

Proof of Proposition 5:

We analyze the case where \bar{Q}_1 is small enough such that $q_2^* > 0$. Substituting for q_1 and q_2 in $P_1 = a - \lambda(q_1 + q_2) + V_R$ we obtain

$$P_1 = \frac{a}{2} + \frac{\lambda \gamma}{2(1 - \gamma)} \bar{Q}_1 + V_R.$$

which is increasing in γ and a . Now consider long-run performance $\left(\frac{V_R - P_1}{P_1}\right)$.

$$\begin{aligned} \text{sign} \left[\frac{\partial}{\partial a} \left(\frac{V_R - P_1}{P_1} \right) \right] &= \text{sign} \left[-\frac{\partial P_1(a)}{\partial a} \frac{V_R}{P_1(a)^2} \right] = -\text{sign} \left[\frac{\partial P_1(a)}{\partial a} \right] \\ \text{sign} \left[\frac{\partial}{\partial \gamma} \left(\frac{V_R - P_1}{P_1} \right) \right] &= \text{sign} \left[-\frac{\partial P_1(\gamma)}{\partial \gamma} \frac{V_R}{P_1(\gamma)^2} \right] = -\text{sign} \left[\frac{\partial P_1(\gamma)}{\partial \gamma} \right] \end{aligned}$$

Thus, long-run performance is decreasing in γ and a .

From Proposition 4, the price reversal ratio is given by

$$\frac{P_1 - P_0}{P_1 - V_R} = \frac{\gamma q_2}{Q} = \frac{\gamma q_2}{q_1 + q_2}.$$

Taking the derivative with respect to a , we get

$$\begin{aligned} \frac{\partial}{\partial a} \left(\frac{P_1 - P_0}{P_1 - V_R} \right) &= \gamma \frac{\partial}{\partial a} \left(\frac{q_2}{q_1 + q_2} \right) \\ &= \frac{\gamma q_1}{(q_1 + q_2)^2} > 0 \end{aligned}$$

The price reversal ratio is not monotonic in γ . To see this, note that $q_2 = 0$ at $\gamma = \frac{a - 2q_1\lambda}{a - q_1\lambda}$.

Thus,

$$\frac{P_1 - P_0}{P_1 - V_R} = \frac{\gamma q_2}{q_1 + q_2} = \begin{cases} 0 & \text{for } \gamma = 0 \\ 0 & \gamma = \frac{a - 2q_1\lambda}{a - q_1\lambda} \end{cases}$$

and $\frac{P_1 - P_0}{P_1 - V_R} > 0$ for $\gamma \in \left(0, \frac{a - 2q_1\lambda}{a - q_1\lambda}\right)$. The derivative of the ratio is

$$\begin{aligned} \frac{\partial}{\partial \gamma} \left(\frac{\gamma q_2}{q_1 + q_2} \right) &= \frac{q_2}{q_1 + q_2} + \gamma \left(\frac{q_1}{(q_1 + q_2)^2} \right) \frac{\partial q_2}{\partial \gamma} \\ \text{sign} \left[\frac{\partial}{\partial \gamma} \left(\frac{\gamma q_2}{q_1 + q_2} \right) \right] &= \text{sign} \left[q_2 (q_1 + q_2) + \gamma q_1 \frac{\partial q_2}{\partial \gamma} \right] \\ &= \text{sign} \left[q_2 (q_1 + q_2) - \frac{\gamma q_1^2}{2(1 - \gamma)^2} \right] \end{aligned}$$

The above is positive at $\gamma = 0$ and negative at $\gamma = \frac{a - 2q_1\lambda}{a - q_1\lambda}$ as $q_2 = 0$. The second derivative is given by

$$\begin{aligned} \frac{\partial^2}{\partial \gamma^2} \left(\frac{\gamma q_2}{q_1 + q_2} \right) &= \frac{q_1}{(q_1 + q_2)^3} \left(2(q_1 + q_2) \frac{\partial q_2}{\partial \gamma} + \gamma (q_1 + q_2) \frac{\partial^2 q_2}{\partial \gamma^2} - 2g \left(\frac{\partial q_2}{\partial \gamma} \right)^2 \right) \\ &< 0 \text{ [as } \frac{\partial q_2}{\partial \gamma} < 0 \text{ and } \frac{\partial^2 q_2}{\partial \gamma^2} = -\frac{q_1}{(1 - \gamma)^3} < 0]. \end{aligned}$$

Thus, there exists a $\hat{\gamma}$ such that for all $\gamma < \hat{\gamma}$, $\frac{\partial}{\partial \gamma} \left(\frac{P_1 - P_0}{P_1 - V_R} \right) > 0$ and for $\gamma > \hat{\gamma}$, $\frac{\partial}{\partial \gamma} \left(\frac{P_1 - P_0}{P_1 - V_R} \right) < 0$.

The first-day return $\left(\frac{P_1 - P_0}{P_0} \right)$ is monotonic in a . To see this, examine

$$P_0 = P_1 - \frac{\gamma q_2}{Q} (P_1 - V_R)$$

$$\frac{P_0}{P_1} = 1 - \frac{\gamma q_2}{Q} \left(1 - \frac{V_R}{P_1} \right)$$

$$\begin{aligned} \text{sign} \left[\frac{\partial}{\partial a} \left(\frac{P_1 - P_0}{P_0} \right) \right] &= -\text{sign} \left[\frac{\partial}{\partial a} \left(\frac{P_0}{P_1} \right) \right] \\ &= -\text{sign} \left[- \left(1 - \frac{V_R}{P_1} \right) \frac{\partial}{\partial a} \left(\frac{q_2}{q_1 + q_2} \right) + \frac{V_R q_2}{q_1 + q_2} \frac{\partial}{\partial a} \left(\frac{1}{P_1} \right) \right] \\ &= \text{sign} \left[\left(1 - \frac{V_R}{P_1} \right) \frac{\partial}{\partial a} \left(\frac{q_2}{q_1 + q_2} \right) - \frac{V_R q_2}{q_1 + q_2} \frac{\partial}{\partial a} \left(\frac{1}{P_1} \right) \right] \\ &= \text{positive} \end{aligned}$$

The above uses the following:

$$\begin{aligned} \frac{\partial}{\partial a} \left(\frac{q_2}{q_1 + q_2} \right) &= \frac{\partial q_2(a)}{\partial a} \frac{q_1}{(q_1 + q_2(a))^2} > 0 \\ \frac{\partial}{\partial a} \left(\frac{1}{P_1} \right) &< 0 \\ \left(1 - \frac{V_R}{P_1} \right) &> 0. \end{aligned}$$

Similarly,

$$\begin{aligned} \text{sign} \left[\frac{\partial}{\partial \gamma} \left(\frac{P_1 - P_0}{P_0} \right) \right] &= -\text{sign} \left[\frac{\partial}{\partial \gamma} \left(\frac{P_0}{P_1} \right) \right] \\ &= \text{sign} \left[\left(1 - \frac{V_R}{P_1} \right) \frac{\partial}{\partial \gamma} \left(\frac{\gamma q_2}{q_1 + q_2} \right) - \frac{V_R \gamma q_2}{q_1 + q_2} \frac{\partial}{\partial \gamma} \left(\frac{1}{P_1} \right) \right] \end{aligned}$$

The second term is positive and so is the first for $\gamma < \hat{\gamma}$. Thus, for low γ the first-day return is increasing in γ .

■

Proof of Proposition 7:

Differentiating the right-hand side of the constraint in (6) with respect to a we obtain

$$\begin{aligned} & \text{sign} \left[\frac{\partial}{\partial a} \gamma q_2^*(a, \gamma) (a - \lambda(q_1^*(a, \gamma) + q_2^*(a, \gamma))) \right] \\ &= \text{sign} \left[(a - \lambda \bar{Q}_1 - 2q_2^* \lambda) \frac{\partial q_2^*}{\partial a} \right] \\ &= \text{positive} \end{aligned}$$

Thus, if the constraint binds for some a , it will also bind for all higher a .

The constraint can bind only if $q_2 > 0$, i.e. $\bar{Q}_1 < \frac{a(1-g)}{\lambda(2-g)}$. Let $\bar{Q}_1 = \beta \frac{a(1-g)}{\lambda(2-g)}$ where $\beta < 1$.

Substituting in the constraint we get

$$\begin{aligned} & \gamma q_2^*(a, \gamma) (a - \lambda(q_1^*(a, \gamma) + q_2^*(a, \gamma))) \\ &= \gamma \left(\frac{a}{2\lambda} - \bar{Q}_1 \left(1 + \frac{\gamma}{2(1-\gamma)} \right) \right) \left(\frac{a}{2} + \bar{Q}_1 \frac{\gamma\lambda}{2(1-\gamma)} \right) \\ &= \gamma \left(\frac{a}{2\lambda} - \beta \frac{a(1-\gamma)}{\lambda(2-\gamma)} \left(1 + \frac{\gamma}{2(1-\gamma)} \right) \right) \left(\frac{a}{2} + \beta \frac{a(1-\gamma)}{\lambda(2-\gamma)} \frac{\gamma\lambda}{2(1-\gamma)} \right) \\ &= \frac{\gamma}{4\lambda} \left(a - \beta \frac{2a(1-\gamma)}{(2-\gamma)} - \beta \frac{a\gamma}{(2-\gamma)} \right) \left(a + \beta \frac{a\gamma}{(2-\gamma)} \right) \end{aligned}$$

Taking the derivative of the above with respect to γ , we obtain

$$a^2 (1 - \beta) \frac{4(1-\gamma) + \gamma^2(1-\beta) + 4\beta\gamma}{(2-\gamma)^2} > 0$$

■

Proof of Proposition 8:

If the constraint binds at (q_1^*, q_2^*) then $q_2^* \geq 0$, which implies that $q_1^* = \bar{Q}_1$. Similar to the argument provided in Proposition 1, it is easy to see that $q_1^c = q_1^* = \bar{Q}_1$. The optimal q_2 if the penalty constraint binds is one of the two solutions to the following quadratic:

$$R = q_2 \gamma (a - \lambda(q_1 + q_2)).$$

Solving for q_2 we obtain

$$q_2 = \frac{1}{2\lambda} \left((a - q_1 \lambda) \pm \sqrt{(a - q_1 \lambda)^2 - \frac{4R\lambda}{\gamma}} \right).$$

Calculating the objective function at the above two solutions, we can show that the difference in value of the objective function at $q_2 = \frac{1}{2\lambda} \left((a - q_1 \lambda) - \sqrt{(a - q_1 \lambda)^2 - \frac{4R\lambda}{\gamma}} \right)$ and the value

of the objective function at $q_2 = \frac{1}{2\lambda} \left((a - q_1\lambda) + \sqrt{(a - q_1\lambda)^2 - \frac{4R\lambda}{\gamma}} \right)$ is $q_1 \sqrt{(a - q_1\lambda)^2 - \frac{4R\lambda}{\gamma}}$, which is positive. Hence, the constrained optimal value of q_2 is the one given in the statement of the proposition. ■

Proof of Proposition 9:

In the proof we use q_1 and q_2 instead of q_1^c and q_2^c , respectively. When R is binding then

$$\gamma q_2 [P_1 - V_R] = R$$

and

$$q_2 = \frac{1}{2\lambda} \left((a - q_1\lambda) - \sqrt{(a - q_1\lambda)^2 - \frac{4R\lambda}{\gamma}} \right).$$

Substituting q_2 in P_1 we get

$$P_1 = \frac{a - q_1\lambda}{2} + \frac{1}{2} \sqrt{(a - q_1\lambda)^2 - \frac{4R\lambda}{\gamma}} + V_R$$

which is decreasing in R .

$$\text{sign} \left[\frac{\partial}{\partial R} \left(\frac{V_R - P_1}{P_1} \right) \right] = -\text{sign} \left[\frac{\partial P_1(a)}{\partial R} \frac{V_R}{P_1(a)^2} \right] = -\text{sign} \left[\frac{\partial P_1(a)}{\partial R} \right]$$

Thus, long-run performance is increasing in R .

Now consider the price reversal ratio:

$$\text{sign} \left[\frac{\partial}{\partial R} \left(\frac{P_1 - \hat{P}_0}{P_1 - V_R} \right) \right] = \text{sign} \left[\frac{\partial}{\partial R} \left(\frac{P_1 - P_0}{P_1 - V_R} \right) + \frac{\partial}{\partial R} \left(\frac{r}{P_1 - V_R} \right) \right]$$

Looking at the two terms separately,

$$\begin{aligned} \text{sign} \left[\frac{\partial}{\partial R} \left(\frac{P_1 - P_0}{P_1 - V_R} \right) \right] &= \text{sign} \left[\frac{\partial}{\partial R} \left(\frac{\gamma q_2}{q_1 + q_2} \right) \right] \\ &= \text{sign} \left[\frac{\gamma q_1}{(q_1 + q_2)^2} \frac{\partial q_2}{\partial R} \right] \\ &= \text{positive} \end{aligned}$$

and

$$\text{sign} \left[\frac{\partial}{\partial R} \left(\frac{r}{P_1 - V_R} \right) \right] = -\text{sign} \left[\frac{\partial P_1}{\partial R} \right] = \text{positive}$$

Thus,

$$\frac{\partial}{\partial R} \left(\frac{P_1 - \hat{P}_0}{P_1 - V_R} \right) > 0$$

To prove the comparative statics on the first-day return, examine

$$\begin{aligned} \text{sign} \left[\frac{\partial}{\partial R} \left(\frac{P_1 - \hat{P}_0}{\hat{P}_0} \right) \right] &= \text{sign} \left[\frac{\partial}{\partial R} \left(\frac{P_1 - P_0 + r}{P_0 - r} \right) \right] \\ &= \text{sign} \left[\frac{\partial (P_1 - P_0)}{\partial R} (P_0 - r) - \frac{\partial P_0}{\partial R} (P_1 - P_0 + r) \right] \\ &= \text{sign} \left[\frac{\partial (P_1 - P_0)}{\partial R} P_0 - \frac{\partial P_0}{\partial R} (P_1 - P_0) - r \frac{\partial P_1}{\partial R} \right] \end{aligned}$$

Given that $\frac{\partial P_1}{\partial R} > 0$, to show $\frac{\partial}{\partial R} \left(\frac{P_1 - \hat{P}_0}{\hat{P}_0} \right) > 0$, it is sufficient to show that

$$\begin{aligned} \frac{\partial (P_1 - P_0)}{\partial R} P_0 - \frac{\partial P_0}{\partial R} (P_1 - P_0) &> 0 \\ \Leftrightarrow \frac{\partial}{\partial R} \left(\frac{P_1 - P_0}{P_0} \right) &> 0 \end{aligned}$$

To show the above, we need the following substitution:

$$\begin{aligned} \frac{P_1 - P_0}{P_1 - V_R} &= \frac{\gamma q_2}{q_1 + q_2} \\ P_1 - P_0 &= \frac{\gamma q_2 (P_1 - V_R)}{q_1 + q_2} \\ &= \frac{R}{(q_1 + q_2)} \end{aligned}$$

Similarly,

$$\begin{aligned}
\text{sign} \left[\frac{\partial}{\partial R} \left(\frac{P_1 - P_0}{P_0} \right) \right] &= \text{sign} \left[\frac{\partial}{\partial R} \left(\frac{R}{P_0 (q_1 + q_2)} \right) \right] \\
&= \text{sign} \left[P_0 (q_1 + q_2) - R \frac{\partial}{\partial R} (P_0 (q_1 + q_2)) \right] \\
&= \text{sign} \left[P_0 (q_1 + q_2) - R \frac{\partial}{\partial R} ((P_0 - V_R) (q_1 + q_2)) - R V_R \frac{\partial q_2}{\partial R} \right] \\
&= -\text{sign} \left[\frac{\partial}{\partial} (P_0 - V_R) (q_1 + q_2) + V_R \frac{\partial q_2}{\partial a} \right] \\
&= -\text{sign} \left[\frac{\partial}{\partial a} \Pi (q_2 (\gamma)) + V_R \frac{\partial q_2}{\partial a} \right] \\
&= -\text{sign} \left[\frac{\partial \Pi}{\partial q_2} \frac{\partial q_2}{\partial a} + V_R \frac{\partial q_2}{\partial a} \right] \\
&= \text{sign} \left[\frac{\partial \Pi}{\partial q_2} + V_R \right] \\
&= \text{positive}
\end{aligned}$$

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