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Market-Based Corrective Actions: The Case of Bank Supervision[1]

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Abstract

Many policy proposals suggest that bank supervision should make use of the market prices of traded bank securities. We study the theoretical underpinnings of these proposals in light of a key problem: if the regulator uses market prices, prices adjust to reflect this use and potentially become less revealing. We demonstrate that these proposals are feasible only when the information gap between the market and the regulator is not too large. Thus, there is a strong complementarity between market information and the regulator's information. We demonstrate that the type of security being traded matters for the observed equilibrium outcome and discuss other policy measures that can increase the ability of regulators to make use of market information. Our insights are not limited to the case of market-based bank supervision, but rather apply to a wide set of situations in financial economics, including shareholder activism, takeovers, and the decision to replace CEOs.

The Minneapolis Fed has been calling for increased market discipline and greater use of market data in the bank supervision process for about 20 years. ... Market data are generated by a very large number of participants. Market participants have their funds at risk of loss. A monetary incentive provides a perspective on risk taking that is difficult to replicate in a supervisory context. Unlike accounting-based measures, market data are generated on a nearly continuous basis and to a considerable extent anticipates future performance and conditions. Raw market prices are nearly free to supervisors. This characteristic seems particularly important given that supervisory resources are limited and are diminishing in comparison to the complexity of large banking organizations.

Gary Stern, President, Federal Reserve Bank of Minneapolis, 2001[1]

The Federal Reserve and other regulatory agencies already monitor subordinated debt yields and issuance patterns in evaluating the condition of large banking organizations. ... This use of subordinated debt is one example of the effort supervisors should undertake to employ data from a variety of markets.

Alan Greenspan, Chairman, Federal Reserve Board, 2001[2]

1 Introduction

Bank supervisors in the U.S. increasingly rely on the market prices of bank securities. As Feldman and Schmidt (2003) and Burton and Seale (2005) empirically document, supervisors already make substantial use of market information. Moreover, as demonstrated by the two statements quoted above, many policy makers call for strengthening the reliance on market data. In the same spirit, an important component of the Basel II reform of international bank capital regulations is to encourage the use of market discipline, one form

[1] See http://www.minneapolisfed.org/pubs/region/01-09/stern.cfm

[2] See: http://www.minneapolisfed.org/pubs/region/01-09/greenspan.cfm

of which is the use of information in market prices.[3] Another frequent proposal, most recently advocated by Evanoff and Wall (2004) and Herring (2004), would require banks to regularly issue subordinated debt, partly so that supervisors could use the price to monitor the health of issuing banks.

The rationale behind market-based supervision is clear. On the one hand, market prices are thought to provide useful information because they aggregate the private information and beliefs of many different people who trade in the financial market.[4] Indeed, several papers in the banking literature document that bank security prices reflect underlying risk and that markets have information that regulators do not have – see, for example, Krainer and Lopez (2004) and the surveys by Flannery (1998) and Furlong and Williams (2006). On the other hand, direct supervision by regulators is expensive and limited. In the United States, federal and state governments spent nearly three billion dollars in 2005 supervising banks and other depository institutions such as thrifts and credit unions. Banks spent five to eight times as much complying with the rules and required procedures. This corresponds to 18-28 basis points of the spread between borrowing and lending rates.[5] Moreover, direct supervision is limited because banks' balance sheets are more complex today than they used to be and because supervisors may not be able to obtain all the relevant information held

[3] More generally, market discipline is the reliance on and use of private counterparty supervision to monitor and limit bank risk. Information produced by market participants and reflected in market prices provides one form of market discipline.

[4] This idea goes back to Hayek (1945), who argues that markets provide an efficient mechanism for information production and aggregation. Empirical evidence has indeed demonstrated the ability of financial markets to produce information that accurately predicts future events (see Roll (1984)), and that this information is being used for real investment decisions (see Luo (2005), Chen, Goldstein and Jiang (2007), and Bakke and Whited (2007)).

[5] In more detail, Elliehausen (1998) surveys several studies that have estimated regulatory costs. He reports estimates ranging from 12-14 percent of non-interest expenses of which 5-8 percent is attributed to safety and soundness regulation. (The rest is attributed to consumer protection regulation.) Using FDIC (2006) numbers, in 2005 the total non-interest expense of all FDIC insured deposit institutions was 317 billion dollars, which gives a regulatory cost in the range of 16-25 billion dollars. In 2005, these institutions had roughly 9 trillion dollars of assets that earned interest and dividend income. Therefore, the estimated regulatory cost accounts for about 18-28 basis point of the interest rate spread.

by other market participants.

A fundamental issue that needs to be considered for market-based supervision is that market prices reflect information, not only about bank fundamentals, but also about the regulator's expected action. In some cases this considerably complicates the inference of information from the price. Suppose that a decrease in bank fundamentals leads the regulator to take a *corrective* action that benefits bank investors and thus increases the price of bank securities. Then, inferring information from the price is a challenge: a moderate price may indicate either that fundamentals are bad and that the regulator is expected to intervene and improve the bank's health, or that fundamentals are not bad enough to justify intervention. Such a situation is empirically and practically quite relevant. Indeed, as we describe in the next section, many actions taken by US regulators are aimed to improve bank health and thus are expected to benefit bank investors. Moreover, DeYoung, Flannery, Lang, and Sorescu (2001) directly show that the price of bank debt increases in response to an unexpectedly poor exam rating for lower quality banks.[6]

In this paper, we characterize the rational-expectations equilibria of a model in which the price of bank debt both affects and reflects the regulator's action. Our focus is on the theoretically challenging – yet empirically relevant – case described above, i.e., where the price exhibits non-monotonicity with respect to the fundamentals due to the positive effect that the regulator's corrective action has on the value of the bank's debt. In this case, learning from the price is complicated by the fact that different fundamentals may be associated with the same price. The equilibrium analysis, in turn, becomes quite challenging given that the price has to reflect the expected regulatory action, which depends on the price in a non-trivial way.

Before describing the results of our analysis, let us explain the relation between our model and existing literature. A key feature of our analysis is that prices in financial markets affect the real value of securities via the information they provide to decision makers. In

[6]Related to this is the finding by Covitz, Hancock, and Kwast (2004) and Gropp, Vesala, and Vulpes (2006) that a weak relation between the market price of debt and risk is observed when the government support of debt holders is more likely.

that, our model is different from the vast majority of papers on financial markets, where the real value of securities is assumed to be exogenous (e.g., Grossman and Stiglitz (1980)). Our paper contributes to a growing literature that analyzes models in which an economic agent seeks to glean information from a market price and then takes an action that affects the value of the security – see, Fishman and Hagerty (1992), Khanna, Slezak, and Bradley (1994), Boot and Thakor (1997), Dow and Gorton (1997), Fulghieri and Lukin (2001), Goldstein and Guembel (2007), Bond and Eraslan (2007), and Dow, Goldstein and Guembel (2007).[7]

The above papers, however, do not consider the main case of interest in our model, where inference from the price is complicated by the fact that one price can be consistent with different fundamentals. Thus, all these other papers are nested as a special case of our model, the analysis of which is summarized in Section 4.1, where the price function is monotone.

Perhaps the only theoretical mention of the problem we focus on here is made by Bernanke and Woodford (1997) in the context of monetary policy. They observe that if the government tries to implement a monetary policy that is based on inflation forecasts, it might lead to non-existence of a rational-expectations equilibrium.[8] Our analysis goes much beyond this basic observation. In particular, by studying a richer model, we are able to demonstrate under what conditions an equilibrium exists, and to characterize the informativeness of the price and the efficiency of the resulting regulatory action when an equilibrium does exist.[9] Thus, we make a first step in analyzing the equilibrium results of a very involved problem, where the use of market data is self defeating in the sense that the reflection of the expected market-based action in the price destroys the informational

[7]See also Subrahmanyam and Titman (1999) and Foucault and Gehrig (2007) for models where the information in the price affects a corporate action, but this is not reflected in the price of the security, and Ozdenoren and Yuan (2007) where the effect of prices on real value is not due to information.

[8]For a similar observation in the context of bank supervision, see the recent working paper by Birchler and Facchinetti (2007).

[9]Other papers study different dimensions of market-based regulation. Faure-Grimaud (2002), Rochet (2004), and Lehar, Seppi, and Strobl (2007) study the effect of market prices on a regulator's commitment ability. Morris and Shin (2005) argue that transparency by the central bank may be detrimental as it reduces the ability of the central bank to learn from the market.

content of the price.

We also wish to stress that the problem we analyze in this paper is by no means limited to the context of bank supervision. Rather, it is a pervasive problem that arises in many contexts, where agents take a corrective action that affects the value of a security and that is influenced by its price. Examples where the non-monotone price function in our paper may arise include the case of directors of a public firm deciding whether to replace the CEO, and the case of investors of a public firm deciding whether to intervene in the firm's business (via shareholder activism, takeover, etc.). We discuss these additional applications of our model in Section 6.[10]

Turning to the results of our equilibrium analysis, we show that a key parameter in the characterization of equilibrium outcomes is the quality of the regulator's own information. When the regulator has relatively precise information, he is able to learn from market prices and implement his preferred intervention rule as a unique equilibrium. When the regulator's information is moderately precise, additional undesirable equilibria exist in which the regulator intervenes either too much, or too little. Interestingly, in this range, the type of equilibrium – i.e., whether there is too much or too little intervention – depends on whether the traded security has a convex payoff (junior debt) or a concave payoff (senior debt). Finally, when the regulator's information is imprecise, he is unable to implement his preferred intervention strategy in equilibrium.

Our analysis generates several normative implications for regulatory policy. First and foremost, we demonstrate that there is a strong complementarity between regulators' direct sources of information and their use of market data. Regulators' direct sources of information are crucial to enable the efficient use of market data. This implication is derived

[10]In the world of regulation and policy making, the problem also arises outside the context of bank supervision. Piazzesi (2005) demonstrates the importance of accounting for the dual relation between monetary policy and market prices in explaining bond yields. Another example is the Sarbanes-Oxley Act of 2002. Section 408 of the act calls for the Securities and Exchange Commission to consider market data – namely, share price volatility and price-to-earnings ratios – when deciding whether to review the legality of a firm's disclosures. A final example is class action securities litigation. Courts in the United States use share price changes as a guide for determining damages (see, e.g., Cooper Alexander (1994)).

despite the fact that our model endows the market with perfect information about the fundamentals. The role of the regulator's own information in our model is thus to enable the regulator to tell whether movements in the price are due to changes in fundamentals or changes in expectations regarding the regulator's own action. Second, we analyze other policy measures that help the regulator implement the efficient market-based intervention policy, even when the information gap between the market and the regulator is not small. These measures include tracking the prices of multiple traded securities, disclosure of the regulator's information ("transparency"), introducing a security that pays off in the event that the regulator intervenes, and taxing security holders to change the effect of the regulator's action on the value of their securities.

Our paper also offers several positive implications that are relevant, not only for market-based regulation, but also for other applications, such as when directors or investors take an action based on market price. These applications have been the subject of wide empirical research trying to detect the relation between market prices and the resulting actions. Our paper suggests that the quality of information of agents outside the financial market (i.e., regulators, directors, investors) and the shape of the security, whose price is observed, are key factors affecting the relation between the price and the resulting action. In addition, we argue that two key features of our theory have to be taken into account in empirical research on market-based intervention. First, if agents use the market price in their intervention decision, there will be dual causality between market prices and the intervention decision. In the context of shareholder activism in closed-end funds, Bradley, Brav, Goldstein, and Jiang (2006) conduct empirical analysis that indeed takes into account this dual causality. Failing to account for the dual causality will produce results that appear as just a weak relation between prices and action. Second, when the information that agents have outside the financial market is not precise enough, our model generates equilibrium indeterminacy, which might make the relation between market prices and intervention more difficult to detect.

The remainder of the paper is organized as follows. In Section 2, we present the model. Section 3 defines an equilibrium in our model. In Section 4, we characterize equilibrium outcomes as a function of the information gap between the market and the regulator. Section 5 studies the policy implications of our model. In Section 6, we discuss other applications of our theory. Section 7 concludes. All proofs are relegated to the Appendix.

2 The model

The model has one bank, a regulator, and a financial market that trades the bank's debt.[11] There are three dates, $t = 0, 1, 2$. At date 0, the price of bank debt is determined in the market. At date 1, the regulator may intervene in the bank's operations. Finally, at date 2, all debt holders (and other security holders) are paid. We now describe the set-up in more detail.

The bank: In the absence of regulatory intervention, the bank's assets generate a gross expected cash flow of θ at date 2. We will often refer to θ as the *fundamental* of the bank. The fundamental θ is stochastic and is realized at date 0. Throughout, we assume that the fundamental θ is drawn uniformly from some interval $[\underline{\theta}, \bar{\theta}]$.

Different types of investors may have claims on the bank's cash flows. These include depositors, debt holders, and equity holders. We will be primarily interested in the value of bank's debt and will elaborate on this below.

The regulator: In the United States, a bank regulator who believes that a bank is performing poorly possesses a variety of mechanisms by which he can attempt to improve the bank's health. These range from encouraging bank management to correct identified problems to formal agreements that restrict capital distributions and management fees, limit bank activities, or even dismiss senior officers or directors.[12] If the regulator believes

[11] We focus on the case where the regulator learns from the price of the bank's debt because this is what most policy proposals refer to. Later in the paper, we address the case of learning from the price of the bank's equity either separately or alongside the price of debt.

[12] As an example of the type of actions that US regulators may take, consider the following 2002 written agreement with PNC bank, which was instigated by accounting irregularities. To ensure that PNC implemented among other things the necessary risk management systems and internal controls, the bank was required to hire an independent consultant to "review the structure, functions, and performance of PNC's management and the board of directors oversight of management activities .... The primary purpose of

the bank is suffering from temporary liquidity problems, he can offer to provide funding at a below-market rate. Under some circumstances, these regulatory actions are even mandated by the prompt corrective action provisions in the Federal Deposit Insurance Corporation Improvement Act of 1991. For more details on actions that US regulators can take see Spong (2000).

We model the regulator as having the opportunity to intervene in the bank's business at date 1. If the regulator intervenes, the bank's date 2 expected cash flow increases by an amount $T(\theta)$.[13] We assume that $T(\theta)$ is weakly decreasing in θ. That is, the benefit from the regulator's intervention is high when the bank's fundamentals are low. This is a natural assumption reflecting the idea that there is more room for improvement when the state is bad. Still, $\theta + T(\theta)$ is increasing in θ, that is, in the presence of intervention, the total expected cash flow available to the bank is increasing in fundamentals.

When deciding whether to intervene, the regulator weighs the cost against the benefit. We assume a fixed cost of intervention C.[14] The benefit of intervention for the regulator is denoted as $V(\theta)$, which is decreasing in θ.[15] Assuming that the benefit of intervention is higher than the cost when fundamentals are very low and lower than the cost when fundamentals are very high, that is, $V(\underline{\theta}) > C > V(\bar{\theta})$, there is a unique $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$ at which the regulator is indifferent between intervening and not intervening, i.e., $V(\hat{\theta}) = C$. For fundamentals below (above) $\hat{\theta}$, a fully informed regulator would strictly prefer to intervene

the [review] shall be to assist the board of directors in the development of a management structure that is adequately staffed by qualified and trained personnel suitable to PNC's needs." (Board of Governors of the Federal Reserve System. Docket No. 02-011-WA/RB-HC. Written Agreement by and between PNC Financial Services Group, Inc., Pittsburgh, Pa., and the Federal Reserve Bank of Cleveland. July 2, 2002.)

[13] Although we model intervention as a binary decision, we do allow for probabilistic intervention. However, since we will also require that the regulator's decision be time-consistent (see below), probabilistic intervention rarely occurs.

[14] This is not necessary for our analysis. The only thing that we will need is that C is not decreasing too fast in θ.

[15] For our purposes, the importance of V and C is that together they summarize the behavior of a fully informed regulator. Appropriate regulation is the subject of a substantial literature, see, e.g., the recent paper of Morrison and White (2005) for one positive theory of bank regulation along with the references cited therein.

(not intervene).

One simple way to think about the regulator's problem is that he is interested in maximizing total surplus. In this case, $V(\theta)$ coincides with $T(\theta)$ and the regulator only wishes to intervene when $T(\theta)$ is greater than C. Another way to think about the regulator's problem is that he is interested in protecting depositors, and thus will intervene only when the probability that the bank will not have enough resources to pay depositors is high.[16] In this case, $V(\theta)$ is clearly different from $T(\theta)$: $V(\theta)$ represents the benefit to depositors from intervention, while $T(\theta)$ is the change in total expected cash flow as a result of intervention. Again, $V(\theta)$ is decreasing in θ, since the probability of bankruptcy is high when fundamentals are low.[17]

The price of bank's debt: An anticipated intervention by the regulator in the bank's business affects the value of the bank's debt. In our model, this debt is traded in the financial market at date 0, and its price reflects the expected equilibrium value. We let $X(\theta)$ denote the value of the bank's debt absent regulatory intervention. Note that $X(\theta)$ is strictly increasing in the fundamental θ.

We denote the expected value of regulatory intervention for debt holders as $U(\theta)$. Intervention affects the value of the security through its effect on the bank's cash flows, i.e.,

$$U(\theta) = X(\theta + T(\theta)) - X(\theta). \tag{1}$$

Information: A key point in our analysis is that the regulator does not know θ, and may learn it from the market price of debt. We assume that the realization of θ is known in the market at date 0, and that it serves as a basis for the price formation. In addition, at date 0, the regulator observes a noisy signal of θ: $\phi = \theta + \xi$. We assume that ξ, the noise with which the regulator observes the fundamental, is uniformly distributed over $[-\kappa, \kappa]$, and that ϕ is not observed by the market.

[16]This possibility was modelled explicitly in a previous version of the paper. Details are available from the authors upon request.

[17]We assume that the type of intervention performed by the regulator cannot be replicated by the banks' security holders. We make this assumption because bank supervisors have legal powers that enable them to control bank actions. Governance by security holders, on the other hand, is limited, as emphasized by the literature on corporate governance.

Robustness: One limitation of our informational structure is that it assumes that the regulator always knows less than the information collectively possessed by market participants (i.e., the information of market participants aggregates to θ, while the regulator only observes a noisy signal of θ). To assess the robustness of the model, we have also analyzed an extension in which the regulator sometimes has more information than the market. We model this by assuming that the bank's cash flows are given by $\theta + \varepsilon$, and the regulator sometimes observes $\theta + \varepsilon$, while the market only observes θ. Thus, while the market always observes θ, the regulator sometimes observes just a noisy signal of θ, and sometimes observes a better signal, which is the actual realization of cash flow $\theta + \varepsilon$. This extension does not affect the qualitative results of the model; details are available upon request.

Finally, in our model the state variable θ captures the expected cash flow of the bank. Intervention by the regulator affects this state variable by changing the expected cash flow by T. It is important to note that both the state variable θ and the effect of intervention T can be interpreted in different ways. In particular, a leading alternative interpretation is that θ is a measure of the riskiness of the bank's cash flows, while regulatory intervention seeks to curb risk-taking behavior. Again, intervention is corrective: the regulator intervenes when the bank is too risky, and a reduction in risk benefits debt holders.

3 Equilibrium

3.1 Market price

When making his intervention decision, the regulator possesses two pieces of information: his own signal ϕ and the observed price of the bank's debt P. An intervention policy is thus a function $I(P, \phi)$, where $I \in [0, 1]$ is the probability of intervention.

For a given intervention policy $I(\cdot, \cdot)$, the price of debt incorporates the intervention probability. Specifically, the equilibrium pricing function P satisfies the rational expectations equilibrium (REE) condition

$$P(\theta) = X(\theta) + E_\phi [I(P(\theta), \phi) | \theta] U(\theta) \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}]. \tag{2}$$

The first component in this expression is the expected value of debt absent regulatory intervention given the fundamental θ. The second component is the additional value stemming from the possibility of regulatory intervention, the probability of which depends on the price $P(\theta)$ and the regulator's own signal ϕ.

3.2 Time consistency

For most of the analysis in the paper (Section 5.5 is the exception), we also require the regulator's intervention policy to constitute a "best response" to the market price. That is, we require the intervention policy to be *time consistent*. This implies that the regulator intervenes with probability 1 when the expected benefit from intervention is greater than the cost and intervenes with probability 0 when the expected benefit is smaller than the cost. Thus, under time consistency, I is either 0 or 1, except for the case in which the expected benefit is exactly equal to the cost. In this case, the regulator may choose to play a mixed strategy.

For a given pricing function $P(\cdot)$, the observation of a particular price P tells the regulator that the market observed a fundamental θ' such that $P(\theta') = P$. Formally, the intervention policy $I(\cdot,\cdot)$ is time consistent given a pricing function $P(\cdot)$, when for all equilibrium realizations $\left(\tilde{P},\tilde{\phi}\right)$ of the price-signal pair,

$$I\left(\tilde{P},\tilde{\phi}\right) = \begin{cases} 1 & \text{if } E_\theta\left[V(\theta)\,|P(\theta) = \tilde{P} \text{ and } \tilde{\phi}\right] > C \\ 0 & \text{if } E_\theta\left[V(\theta)\,|P(\theta) = \tilde{P} \text{ and } \tilde{\phi}\right] < C \end{cases}. \tag{3}$$

(Note that if the expected value of intervention exactly equals the cost C, any probability of intervention is time consistent.)

3.3 Equilibrium definition

The formal definition of an equilibrium is as follows:

Definition 1 *A pricing function $P(\cdot)$ and an intervention policy $I(\cdot,\cdot)$ together constitute an equilibrium if they satisfy the REE condition (2) and the time-consistency condition (3).*

4 Market-based intervention

We now explore the equilibrium outcomes when the regulator attempts to learn the fundamental θ from the price of the bank's debt in the financial market.

We start by defining an important class of equilibria for which the regulator can perfectly infer the market's information.

Definition 2 *A fully revealing equilibrium is an equilibrium in which each price is associated with one fundamental, and thus the fundamental can be inferred from the price. An equilibrium is essentially fully revealing if when a price is associated with more than one fundamental the regulator can still distinguish among the different fundamentals based on his signal.*

In both fully revealing and essentially fully revealing equilibria, the regulator chooses the optimal action based on θ: he intervenes when θ is less than the cutoff value $\hat{\theta}$ and does not intervene when θ is above $\hat{\theta}$. Thus, we will often refer to fully revealing and essentially fully revealing equilibria as equilibria with *optimal intervention.* Clearly, any other equilibrium does not feature optimal intervention since in such equilibrium, the combination of the price and the regulator's signal does not enable the regulator to infer θ perfectly.

From (1), the price function for the debt security under optimal intervention is given by

$$P(\theta) = \begin{cases} X(\theta + T(\theta)) & \text{if } \theta < \hat{\theta} \\ X(\theta) & \text{if } \theta > \hat{\theta} \end{cases}. \tag{4}$$

The main question we are interested in is whether optimal intervention is an equilibrium outcome, and if it is, then whether it is the unique equilibrium outcome.

4.1 Monotone price function: $T\left(\hat{\theta}\right) \leq 0$

We start with a simple case where optimal intervention is the unique equilibrium outcome, independent of the accuracy of the regulator's signal. This happens when intervention at $\hat{\theta}$ reduces the bank's expected cash flow, i.e., $T\left(\hat{\theta}\right) \leq 0$. Some forms of intervention indeed fall within this case. One example is a firesale liquidation of bank assets. Here, the regulator

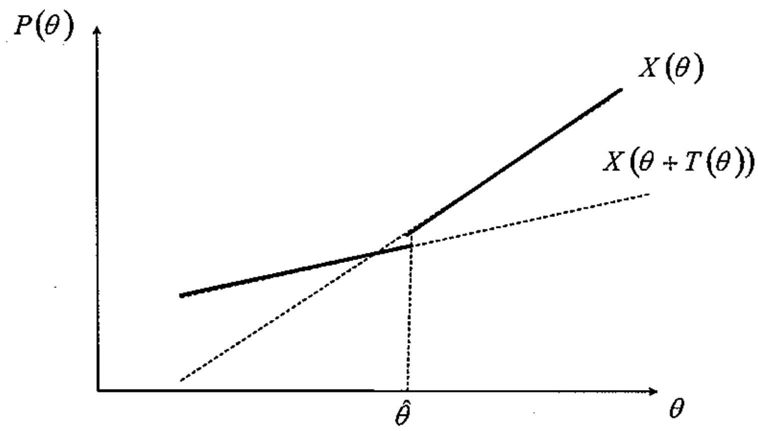

Figure 1: Debt price under optimal intervention when $T\left(\hat{\theta}\right) < 0$

liquidates in order to ensure payment to depositors. This, however, reduces the cash flows to other claim holders – and in particular to debt holders who are our main focus – and thus the value of their securities declines. The formal result for this case is in Proposition 1.

Proposition 1 *If* $T\left(\hat{\theta}\right) \leq 0$ *then (for all regulator signal accuracies* κ*) an equilibrium with optimal intervention exists, and is the unique equilibrium.*

To see the intuition behind this result, it is useful to inspect Figure 1, which displays the price function (4) for this case. (Note that Figure 1 and the other figures in the paper are only schematic. In particular, the functions are drawn as linear functions, although they need not be linear.) In the figure we see the price of debt under intervention – $X(\theta + T(\theta))$ – and the price under no intervention – $X(\theta)$. The regulator wishes to intervene if and only if $\theta < \hat{\theta}$, and thus the optimal intervention for him generates a price function which is depicted by the bold lines in the figure. The key property of this function is that it is monotone in θ. Hence, every level of the fundamental θ is associated with a different price. This implies that the regulator can learn the realization of θ precisely from the price and thus act optimally, regardless of how imprecise his signal is.

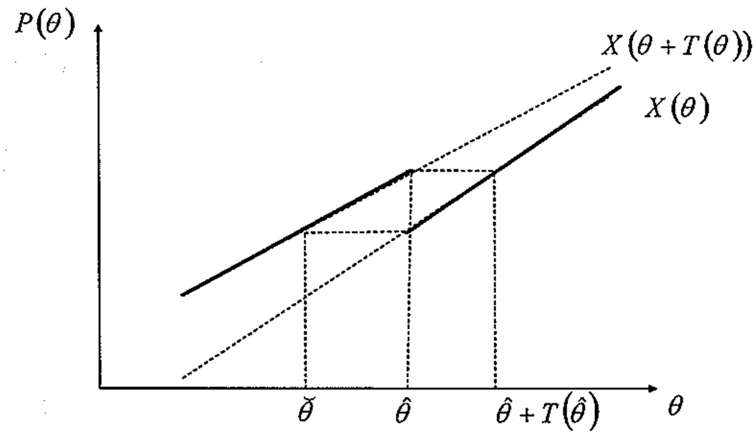

Figure 2: Debt price under optimal intervention when $T\left(\hat{\theta}\right) > 0$

This case of a monotone price function is the one analyzed in the existing literature on the feedback effect from asset prices to the real value of securities (see the description in the introduction). We now turn to the case which is the focus of our analysis – that of a non-monotone price function.

4.2 Non-monotone price function: $T\left(\hat{\theta}\right) > 0$

In most situations things are not so simple as described in the previous section. The aim of regulation is in many cases to improve the health of the bank, and thus $T\left(\hat{\theta}\right)$ is likely to be greater than 0. This is indeed consistent with most types of intervention mentioned in Section 2, and with empirical evidence discussed below. In the rest of the paper, we thus focus on this more interesting case. Figure 2 displays the price function (4) when $T\left(\hat{\theta}\right) > 0$.

Inspection of Figure 2 reveals the difficulty in obtaining an equilibrium with optimal intervention when $T\left(\hat{\theta}\right) > 0$. The difficulty stems from the fact that under optimal intervention, the price function is non-monotone and that the non-monotonicity occurs around $\hat{\theta}$. That is, when the fundamental decreases and crosses the threshold $\hat{\theta}$, the regulator wishes to intervene. Intervention, in turn, increases the value of the bank's debt (because

$T\left(\hat{\theta}\right) > 0$), which implies that the price of debt is non-monotone with respect to the fundamentals. This feature is consistent with empirical evidence provided by DeYoung, Flannery, Lang, and Sorescu (2001), who show that the price of bank debt increases in response to an unexpectedly poor exam rating for lower quality banks. It is also consistent with Covitz, Hancock, and Kwast (2004) and Gropp, Vesala, and Vulpes (2006), who show that a weak relation between the market price of debt and risk is observed when the government support of debt holders is more likely. While in our model non-monotonicity arises in part from the discreteness of the intervention decision, it is important to note that this feature is certainly not necessary for non-monotonicity. Indeed, Birchler and Facchinetti (2007) recently show that, as long as there is some fixed cost in intervention, non-monotonicity will be a feature of the price function even if the intervention decision is continuous.

The implication of the non-monotonicity in the price of debt under optimal intervention is that fundamentals on both sides of $\hat{\theta}$ have the same price. In particular, there is a range of fundamentals between $\check{\theta} \equiv \hat{\theta} - T\left(\check{\theta}\right)$ and $\hat{\theta} + T\left(\hat{\theta}\right)$, where, under optimal intervention, each fundamental has the same price as another fundamental. This implies that the regulator can infer neither the level of the fundamental, nor the optimal action, from the price alone. Essentially, the fact that the price reflects the expected reaction of the regulator to the price makes learning from the price more difficult. A natural conjecture that follows from this discussion is that the possibility of achieving optimal intervention in equilibrium depends on the precision of the regulator's signal. A precise signal will enable the regulator to distinguish between different fundamentals that have the same price. We thus turn to provide a complete analysis of equilibrium outcomes based on the precision of the regulator's signal.

As it turns out, the shape of the value of bank debt $X(\theta)$ with respect to θ is an important factor for the characterization of equilibrium outcomes. Thinking about the typical financial structure of banks, debt securities are usually convex for low fundamentals and concave for high fundamentals. Economically, the convex then concave shape arises because debt is junior to deposits but senior to equity claims. In our model, we are able to characterize equilibrium outcomes for either a concave or a convex $X(\cdot)$ function. That is, we can characterize results for situations where the relevant fundamentals (i.e., some range around $\hat{\theta}$) are either in the range where debt is concave or in the range where debt is convex. To keep the exposition short, we will present only the results for a concave debt security (which implies that we are looking at a relatively senior debt security).[18]

4.2.1 The regulator's signal is precise: efficient equilibrium

We start with the case in which the regulator's signal ϕ is relatively precise. Our first result is:

Proposition 2 *For $\kappa < T\left(\hat{\theta}\right)/2$, an equilibrium with optimal intervention exists.*

The intuition behind this result is as follows. Under the optimal intervention rule, there are at most two fundamentals associated with each price. Suppose that θ_1 and θ_2, $\theta_1 < \hat{\theta} < \theta_2$, have the same price. Under optimal intervention, these fundamentals are at a distance $T(\theta_1)$ from each other (see Figure 2). Since the regulator's signal is relatively precise, the regulator can use the signal to perfectly infer the realization of the fundamental when the price is consistent with two different fundamentals at a distance $T(\theta_1)$ from each other. Thus, he can follow the optimal intervention rule. It is worth stressing that in this equilibrium both the price and the signal serve an important role: the price tells the regulator that one of two different fundamentals may have been realized, while the signal enables the regulator to differentiate between these two fundamentals. Thus, the regulator uses both the price and the signal to infer the underlying fundamental.

Our next result shows that if the regulator's signal is sufficiently accurate, whenever two fundamentals have the same equilibrium price, the regulator's signal is sufficient to distinguish them. As such, the optimal intervention equilibrium is the only equilibrium. Although intuitive, the proof of this result is involved. The key difficulty is the need to rule out equilibria in which there are an infinite number of fundamentals associated with the same price.

[18] Full characterization of the equilibrium outcomes for a convex debt security is available from the authors upon request.

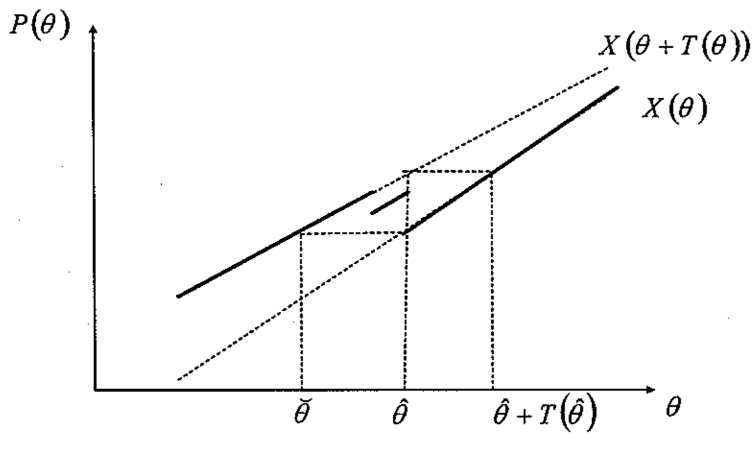

Figure 3: Debt price in an equilibrium with too little intervention

Proposition 3 *There exists $\bar{\kappa} > 0$ such that when $\kappa \leq \bar{\kappa}$ the optimal intervention equilibrium is the unique equilibrium.*

4.2.2 The regulator's signal is moderately precise: inefficient equilibria

As the regulator's information precision worsens, in the sense that κ increases beyond the point $\bar{\kappa}$ defined by Proposition 3 but remains below $T\left(\hat{\theta}\right)/2$, the optimal intervention equilibrium remains an equilibrium. However, additional and less desirable equilibria emerge. In such equilibria, the regulator cannot perfectly infer the fundamental from market prices, and efficient intervention is not obtained. In general, equilibria with non-efficient intervention can either entail too much intervention or too little intervention. As we will establish, whether the equilibrium has too little or too much intervention depends on whether the expected security payoff X is concave or convex. In the case of a concave security, which is our focus, equilibria feature too little intervention. Figure 3 depicts an example of such an under-intervention equilibrium.

In the equilibrium depicted in Figure 3, the regulator intervenes optimally at fundamentals associated with the left line and the right line of the pricing function, but intervenes too little at fundamentals associated with the middle line. These fundamentals are below

$\hat{\theta}$, yet, in the equilibrium, the regulator intervenes with probability less than 1 when they are realized. This happens because every fundamental associated with the middle line has a price that is identical to that of a fundamental associated with the right line. Since the middle line and the right line are close, the regulator cannot always tell apart fundamentals associated with these two lines even after observing his own information. Since fundamentals associated with the middle line are below $\hat{\theta}$ and fundamentals associated with the right line are above $\hat{\theta}$, the regulator does not get clear-cut information as to whether he should intervene or not. Thus, sometimes when the fundamental falls in the middle line, the regulator does not have enough evidence to justify intervention, and chooses not to intervene.

Let us illustrate mathematically what is needed for this equilibrium to hold. Take a pair of fundamentals associated with the middle line and right line of Figure 3 that have the same price, and call them θ_1 and θ_2, respectively. The probability of intervention at θ_2 is 0, and thus the price at θ_2 is $X(\theta_2)$. The probability of intervention at θ_1 is the probability that the regulator observes a signal that is inconsistent with θ_2 conditional on the state of the world being θ_1. Given the uniform distribution of noise that we assumed, this probability is equal to $\frac{\theta_2-\theta_1}{2\kappa}$. Hence, the price at θ_1 is $\left(1-\frac{\theta_2-\theta_1}{2\kappa}\right)X(\theta_1)+\frac{\theta_2-\theta_1}{2\kappa}X(\theta_1+T(\theta_1))$. For the equilibrium to hold, the prices at θ_1 and θ_2 have to coincide, and the regulator's optimal decision when he cannot distinguish between θ_1 and θ_2 has to be not to intervene. Proposition 4 establishes the existence of equilibria of this kind and provides a full characterization of them. The reader can better understand the proposition by inspecting Figure 3 in parallel.

Proposition 4 *Suppose the regulator observes the price of a concave security. For $\kappa < T\left(\hat{\theta}\right)/2$ sufficiently close to $T\left(\hat{\theta}\right)/2$, there exist equilibria with too little intervention. In these equilibria the regulator intervenes with probability less than 1 at some fundamentals below $\hat{\theta}$, and intervenes optimally at other fundamentals. The equilibria differ from one another in the probability of sub-optimal intervention.*

We explained above why under-intervention equilibria, as described in Proposition 4, exist. It is also interesting to explore the source of multiplicity, i.e., why, when κ is in an intermediate range, both optimal intervention (depicted in Figure 2) and under-intervention (depicted in Figure 3) form an equilibrium. Recall that the optimal intervention case constitutes an equilibrium because when intervention is optimal, fundamentals that have the same price are far enough from each other, and so the signal of the regulator, having an intermediate level of precision, is precise enough to enable the regulator to tell the fundamentals apart and intervene optimally. But, suppose that the regulator intervenes with probability less than 1 at some fundamentals that are slightly below $\hat{\theta}$ (as in Figure 3). The lower intervention probability reduces the price at these fundamentals, and creates a situation where fundamentals that are closer to each other have the same price. This then becomes self-enforcing and leads to an equilibrium: as the distance between fundamentals with the same price shrinks, the regulator (with a signal of intermediate precision) cannot always tell these fundamentals apart, and thus intervenes with probability less than 1 at some fundamentals below $\hat{\theta}$.

Another interesting question is whether equilibria with too much intervention exist in parallel to the equilibria with too little intervention identified in Proposition 4. The following proposition provides a negative answer to this question. When the regulator learns from the price of a concave security, any equilibrium with suboptimal intervention entails too little intervention in the following sense:

Proposition 5 *Suppose the regulator observes the price of a concave security, and* $\kappa < T\left(\hat{\theta}\right)/2$. *Then any equilibrium other than the optimal intervention equilibrium entails an intervention probability strictly less than* 1 *at some fundamental* $\theta < \hat{\theta}$.

At first, this result seems surprising. Taking the logic for the presence of multiple equilibria described in the paragraph after Proposition 4, it seems straightforward to apply it in the other direction and generate an equilibrium with too much intervention. But, one has to remember that the presence of a force that pushes towards under- or over-intervention is not enough to guarantee that such an equilibrium will indeed exist. In fact, the equilibria with suboptimal intervention in our model are quite subtle. In particular, as we discussed above, for the equilibria in Proposition 4 to exist the equation

$$X(\theta_2) = \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) X(\theta_1) + \frac{\theta_2 - \theta_1}{2\kappa} X(\theta_1 + T(\theta_1)) \tag{5}$$

has to hold for every pair of θ_1 and θ_2, associated with the middle line and right line, respectively, that have the same price. Hence, it is not that surprising that the set of possible equilibria depends on the shape of the security.

The result in Proposition 5 indicates that while under-intervention equilibria exist for a concave security when the signal of the regulator is intermediately précise, over-intervention equilibria do not exist. As mentioned above, we have also analyzed a model where the regulator learns from the price of a convex security (i.e., junior debt or even equity). Our analysis there established that with a convex security, when the precision of the regulator's signal is in an intermediate range, there exist equilibria with too much intervention, but not with too little intervention.

To understand the role of concavity vs. convexity, let us inspect equation (5), which has to hold in an under-intervention equilibrium. When $X(\theta)$ is concave, this equation implies that

$$\theta_2 < \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) \theta_1 + \frac{\theta_2 - \theta_1}{2\kappa} (\theta_1 + T(\theta_1)). \tag{6}$$

This inequality implies $T(\theta_1) > 2\kappa$, which is consistent with the intermediate range of κ, as defined in Proposition 4. On the other hand, under convexity, equation (5) would imply the opposite, which is, of course, inconsistent. Hence, under-intervention can constitute an equilibrium with a concave security, but not with a convex security. Parallel arguments show that over-intervention can constitute an equilibrium with a convex security, but not with a concave security.

4.2.3 The regulator's signal is imprecise: no equilibrium

Finally, consider the case in which $\kappa > T\left(\hat{\theta}\right)/2$, that is, the regulator's signal is imprecise and the information gap between the market and the regulator is large. The first thing to note is that when $\kappa > T\left(\hat{\theta}\right)/2$, optimal intervention cannot occur in equilibrium. To see this, look again at Figure 2. As we can see in the figure, in an equilibrium with optimal intervention there are fundamentals at a distance of $T\left(\hat{\theta}\right)$ from each other on both sides of $\hat{\theta}$ that have the same price. Since the regulator's signal is imprecise, i.e., since $2\kappa > T\left(\hat{\theta}\right)$, the signal does not enable the regulator to always distinguish between two fundamentals that have the same price. Thus, given a price that is associated with two fundamentals, it is impossible for the regulator to always intervene at one fundamental and never intervene at the other and therefore optimal intervention cannot occur.

Our main result in this subsection is in fact much stronger. Proposition 6 shows that when $\kappa > T\left(\hat{\theta} - 2\kappa\right)/2$, not only is there no equilibrium with optimal intervention, but there is also no other rational-expectations time-consistent equilibrium.

Proposition 6 *When $\kappa > T\left(\hat{\theta} - 2\kappa\right)/2$, no equilibrium exists.*

Although the proof of Proposition 6 is long and involved, in the limiting case in which the regulator receives no information at all (i.e., $\kappa \to \infty$) it is possible to give the following straightforward and intuitive proof. First, we claim that the only candidate equilibrium in this case is one with fully revealing prices. To see this, suppose instead that there is an equilibrium in which two fundamentals θ_1 and $\theta_2 \neq \theta_1$ are associated with the same price. Since the regulator has no information, his intervention policy must be the same at θ_1 and θ_2. But then the prices are not equal, giving a contradiction. (It is important to note that both Proposition 6 and this simple limit argument cover mixed strategies by the regulator.) However, there is no fully revealing equilibrium either: given time-consistency, a fully revealing equilibrium features optimal intervention, a possibility ruled out by the text preceding Proposition 6.

Interpreting the no-equilibrium result No-equilibrium results may seem difficult to interpret. After all, if taken literally, a no-equilibrium result implies that the model cannot predict an outcome. Clearly, the fact that our model generates a no-equilibrium result is due to the rational-expectations equilibrium concept used in the paper. In a fully specified trading game, the no-equilibrium outcome can be translated into an equilibrium with a break-down of trade. This is an equilibrium where in some circumstances market makers abstain from making markets because they would lose money from doing so. We now formalize this interpretation by studying the equilibria of a very simple trading game.

The trading game is as follows. There is a single market maker and multiple speculators. All trade must take place via the market maker. Both the speculators and the market maker observe the fundamental θ. As before, the regulator observes only $\theta + \xi$. After observing the realization of θ, the market maker sets a price, at which he is willing to buy or sell any quantity desired by speculators. The market maker can also abstain from posting a price, in which case no trade takes place. If the market maker posts a price, speculators then submit buy and sell orders. The regulator observes the price set by the market maker and makes an intervention decision just as before.[19]

Clearly this trading game is highly stylized. Its virtue, however, is that it both replicates a rational-expectations equilibrium when one exists, and formalizes the notion that when the regulator's information is poor the market maker abstains from posting a price and trade breaks down. Formally:

Proposition 7 *(A) Let $(P(\theta), I(P(\theta), \phi))$ be a REE. Then there is an equilibrium of the trading game in which for all fundamentals θ and all regulator signal realizations ϕ, the market maker posts price $P(\theta)$ and intervention takes place with probability $I(P(\theta), \phi)$. Conversely, any equilibrium of the trading game with prices posted in all states corresponds to a REE.*

(B) When $\kappa > T\left(\hat{\theta} - 2\kappa\right)/2$, there exists $\theta^* \in (\check{\theta}, \hat{\theta}]$[20] such that for any $\tilde{\theta} \in \left[\check{\theta}, \theta^*\right]$ there is an equilibrium of the trading game in which: the market maker posts the price $X(\theta + T)$ and the regulator intervenes when $\theta \leq \tilde{\theta}$; the market maker does not post a price when $\theta \in (\tilde{\theta}, \tilde{\theta} + T\left(\tilde{\theta}\right)]$; the market maker posts the price $X(\theta)$ and the regulator does not intervene when $\theta > \tilde{\theta} + T\left(\tilde{\theta}\right)$. The equilibria entail suboptimal intervention (except for when $\tilde{\theta} = \check{\theta}$ or $\hat{\theta}$).

Part (A) of Proposition 7 follows almost immediately from definitions. Part (B) is most easily illustrated when the regulator has no information, i.e., $\kappa = \infty$, since this avoids the

[19] If the market maker does not set a price this too is observed by the regulator.

[20] Recall that $\check{\theta}$ is defined by $\check{\theta} + T(\check{\theta}) = \hat{\theta}$.

need to consider off-equilibrium-path beliefs (which are dealt with in the proof). In this case, for any $\tilde{\theta}$ such that $\hat{\theta} \in \left[\tilde{\theta}, \tilde{\theta} + T\left(\tilde{\theta}\right)\right]$ there is an equilibrium in which the market maker posts no price when the fundamental lies in this range, and posts a fully revealing price otherwise. The key property of this equilibrium is that for fundamentals θ in the no-price interval, *any* price that the market maker could conceivably quote would lead to losses. Specifically, in equilibrium, prices above (respectively, below) $X\left(\tilde{\theta} + T\left(\tilde{\theta}\right)\right)$ reveal that the fundamental is above (respectively, below) $\hat{\theta}$ and lead to no intervention (respectively, intervention). So if at fundamental $\theta \in (\tilde{\theta}, \tilde{\theta} + T\left(\tilde{\theta}\right)]$ the marker maker posts a high price, the regulator will respond by not intervening, implying that the quoted price exceeds the fundamental value of the security. In this case speculators short the security and the market maker suffers losses. Likewise, quoting a low price leaves speculators with a profitable buying opportunity.

Several features of this equilibrium are worth commenting upon. First, the equilibrium captures the idea the regulator's action is hard to predict. That is, when the fundamental is in the neighborhood of $\hat{\theta}$, market participants are reluctant to trade at any price, because they do not know how the regulator will react.

Second, unless $\hat{\theta} = \tilde{\theta}$ or $\tilde{\theta} + T\left(\tilde{\theta}\right)$, the equilibrium features suboptimal intervention. To see this, simply note that since the regulator has no information in the above example, he must make the same intervention decision for all fundamentals in the no price range. Since the no price range straddles $\hat{\theta}$, intervention is optimal at some fundamentals in this range but not others. So whatever decision the regulator makes upon seeing no price, it is suboptimal in some cases.

Third, although the fundamental is not fully revealed in equilibrium, the regulator does learn something from the drop in volume that occurs when $\theta \in (\tilde{\theta}, \tilde{\theta} + T\left(\tilde{\theta}\right)]$ — specifically, that the fundamental is in this interval. Indeed, in the extreme equilibria in which $\hat{\theta} = \tilde{\theta}$ or $\tilde{\theta} + T\left(\tilde{\theta}\right)$ this information is enough to allow the regulator to intervene optimally.

5 Policy implications for market-based intervention

The results in the previous section demonstrate the difficulty in implementing an intervention policy that is based on the market price of a bank's security. The problem stems from the fact that the market price adjusts to reflect the expected regulator's action, and this reduces the extent to which the fundamental, which is what the regulator is trying to learn, can be inferred from the price.

A key determinant of whether optimal intervention can be implemented based on market price is the quality of the signal that is observed by the regulator in addition to the price. We show that when the regulator has a very precise signal, optimal intervention is obtained as a unique equilibrium. When the regulator has a moderately precise signal, optimal intervention is still an equilibrium, but there are also other equilibria with suboptimal intervention. Finally, when the regulator's signal is imprecise, there is no rational-expectations equilibrium in the model. We showed that the non-existence of a rational-expectations equilibrium in this range can be interpreted as a market breakdown.

Overall, these results imply that there is a strong complementarity between the market's information and the regulator's information. To be able to implement a successful market-based intervention policy, the regulator still needs to produce a reasonably precise signal of his own. Thus, market-based intervention cannot perfectly substitute for direct supervision but instead is a complement. This is perhaps the main policy implication of our model. It should be noted that this implication is somewhat surprising given that it is obtained despite the fact that our model endows the market with perfect information about the fundamentals of the bank. The role of the information in our model is to help the regulator tell the extent to which the market price reflects information about the fundamental and the extent to which it reflects information about the expected regulator's action. In that sense, the private information in our model plays a somewhat unusual role.

We next study whether there are alternatives to the regulator generating a precise signal for which market-based intervention will work. The first alternative we consider is for the regulator to learn from the prices of multiple securities. The second alternative is to improve transparency by disclosing the regulator's signal to the market. The third alternative is to issue a security that directly predicts whether the regulator is going to intervene. The fourth alternative is to impose taxes on security holders that change their payoffs in case of regulatory intervention. We show that each policy measure ameliorates the regulator's inference problems — although as we describe below, non-trivial conditions must be met for each measure to be feasible in the first place. Finally, we consider the possibility that the regulator can commit ex ante to a policy rule based on the realized price. We show that this does not resolve the regulator's inference problems.

5.1 Multiple securities

Thus far we have restricted attention to the case in which the regulator observes only one price, that of a concave security. We mentioned that parallel results hold for the case in which the regulator observes the price of a convex security instead of that of a concave security. The only difference between the two cases is that in the range of multiple equilibria, underintervention is possible with a concave security, while overintervention is possible with a convex security. A key question is whether it helps if both these securities trade publicly, and the regulator learns from the prices of both.

It turns out that observing the prices of both securities resolves the problem of multiple equilibria when the regulator's signal is moderately precise, but does not solve the problem of no rational-expectations equilibrium when the regulator's signal is imprecise. We start by proving the first result.

Proposition 8 *Suppose that $\kappa < T\left(\hat{\theta}\right)/2$ and that the regulator observes the price of both a strictly concave and a strictly convex security. Then the optimal intervention equilibrium is the unique equilibrium.*

To gain intuition for this result, recall the results of the previous section. There, we showed that when the regulator's information is moderately precise, there may exist equilibria with too much or too little intervention, in addition to the equilibrium with optimal intervention. We also showed that an equilibrium with too much intervention requires that the security whose information the regulator observes be convex, while an equilibrium with too little intervention requires that the security be concave. Thus, in this range, observing both the price of a concave security and the price of a convex security eliminates the equilibria with suboptimal intervention and leaves the optimal intervention equilibrium as the unique equilibrium.

This result suggests that there is a significant benefit to learning from two different securities. Thus, bank regulators can be instructed to learn simultaneously from the prices of bank debt and equity, instead of just from the price of bank debt. It is important to note that this implication of the model requires that two distinct securities trade in well-functioning markets. This condition is not always easily satisfied. In addition, even if this condition is met, the regulator still faces inference problems when his information is imprecise. Specifically, even multiple security prices do not help the regulator when $\kappa > T\left(\hat{\theta} - 2\kappa\right)/2$. The basic intuition utilizing the limiting case in which $\kappa \to \infty$ is the same as that provided for Proposition 6.

Proposition 9 *Suppose that $\kappa > T\left(\hat{\theta} - 2\kappa\right)/2$ and that the regulator observes the price of both a concave and a convex security. Then no equilibrium exists.*

5.2 Transparency

We now return to the case of one traded concave security and assume that the regulator makes public his own signal ϕ *before* the market price is formed. Our analysis implies that this form of transparency improves the regulator's ability to make use of market information. Specifically, transparency resolves the problem of multiple equilibria when the regulator's signal is moderately precise, but it does not solve the problem of no rational-expectations equilibrium when the regulator's signal is imprecise. The argument is as follows.

Under the "transparency" regime in which the regulator truthfully announces his signal ϕ, the equilibrium pricing function depends on both the fundamental θ and the regulator's signal ϕ. Consider a specific realization ϕ^* of the regulator's signal, along with any pair of fundamentals θ_1 and θ_2 such that ϕ^* is possible after both. The prices at (θ_1, ϕ^*) and (θ_2, ϕ^*) must differ. If, instead, the prices coincided, the intervention decisions would also coincide, but in this case the prices would not be equal after all. It follows that all fundamentals θ for which the regulator's signal ϕ^* is possible must have different prices given realization ϕ^*, that is, given ϕ^* prices are fully revealing. This argument together with time-consistency implies that the only candidate equilibrium features optimal intervention. As such, transparency eliminates the suboptimal equilibria of Proposition 4. The intuition is that these equilibria were based on the market not knowing the regulator's action, a problem that is solved once the regulator discloses his signal truthfully.

Now, when $\kappa < T\left(\hat{\theta}\right)/2$, optimal intervention is indeed an equilibrium, with prices $P(\theta,\phi) = X(\theta) + U(\theta)$ for $\theta \leq \hat{\theta}$ and $P(\theta,\phi) = X(\theta)$ for $\theta > \hat{\theta}$. On the other hand, when $\kappa > T\left(\hat{\theta}\right)/2$, optimal intervention is not an equilibrium. To see this, if we suppose to the contrary that it was an equilibrium, then there exist fundamentals θ_1 and θ_2 and a regulator's signal realization $\phi \in [\theta_1 - \kappa, \theta_1 + \kappa] \cap [\theta_2 - \kappa, \theta_2 + \kappa]$ such that (θ_1, ϕ) and (θ_2, ϕ) have the same price, in contradiction to above. It follows that for $\kappa > T\left(\hat{\theta}\right)/2$, there is no equilibrium.

Although a policy of transparency improves the regulator's ability to infer bank fundamentals from market prices, in practice there may be limits to its viability. In particular, if a bank knows that the regulator will make its information public, it may be less inclined to grant easy access to the regulator in the first place. In this sense, it is possible that transparency would serve to increase κ, potentially making the regulator's inference problem worse instead of better.

5.3 Regulator security

Neither of the measures discussed so far allows the regulator to infer the bank's fundamental when his own information is poor ($\kappa > T\left(\hat{\theta} - 2\kappa\right)/2$). The next possibility we discuss is the creation of an "event market" in which market participants trade a security that pays 1 if the regulator intervenes, and 0 otherwise. We will refer to such a security as a *regulator security.* Clearly such a market is feasible only if the regulator's intervention is publicly observable and verifiable — a condition that is not required in any of our analysis to this point, and in practice may fail to hold. However, if such a market could be created, its existence would render optimal intervention as the unique equilibrium *irrespective of the quality of the regulator's information.*

More formally, suppose that in addition to a standard security market, an event market of the type described is feasible and exists. Let Q be the price of the regulator security, with P being the price of the debt security as before. The regulator's intervention policy I can now depend on Q in addition to P and his own signal ϕ. The rational-expectations equilibrium pricing condition for the regulator security is $Q(\theta) = E_{\phi}\left[I\left(P\left(\theta\right), Q\left(\theta\right), \phi\right)|\theta\right]$. Under these conditions we obtain:

Proposition 10 *If the market trades both a standard debt security and the regulator security, then for all κ the unique equilibrium of the economy features optimal intervention.*

The intuition behind this result is the following: a regular debt security may have the same price for different fundamentals because the probability of intervention is different across these fundamentals. But, once the regulator security is traded, the probability of intervention can be inferred from its price, and thus the fundamental can be inferred from the combination of its price and the price of the debt security. This implies that the regulator will intervene optimally in equilibrium.

5.4 Taxation

The next policy tool we consider is taxation. The discussion here is straightforward and relies on our previous results. The problem we discussed in the bulk of the paper – see Section 4.2 – originates from the fact that the price of the security under optimal intervention is non-monotone in θ, and so does not fully reveal θ. This non-monotonicity originates from the fact that $T\left(\hat{\theta}\right) > 0$. In Section 4.1, we showed that if the type of intervention is such that $T\left(\hat{\theta}\right) \leq 0$, then the price function is monotone, and the unique equilibrium indeed features optimal intervention.

In principle, the regulator could achieve monotonicity in the price function, and hence optimal intervention, with taxation. Suppose that the regulator charges the debt holders in case of intervention. This will reduce the value of debt when the regulator intervenes. It is easy to construct the tax scheme in a way that will generate the price function depicted in Figure 1. Given the result in Proposition 1, this will guarantee an equilibrium with optimal intervention.

It is important to note that, relative to the other measures discussed in this section, imposing an intervention-dependent tax on bank security holders is a more drastic policy response. Taxation of this form clearly has distributional consequences and is likely to be costly to implement — both in terms of direct costs, and in terms of other distortions. Moreover, intervention-contingent taxation is possible only if intervention is publicly observable — under which circumstances a regulator security of the type discussed above would also solve the regulator's inference problem.

5.5 Commitment

Thus far in the paper we have assumed that the regulator's intervention decision is ex post efficient, i.e., the regulator does what is optimal for him to do given the prices and his signal. A natural question is whether the regulator can achieve optimal intervention by committing ex ante to a policy rule as a function of the realized price. To answer this question, we assume that the regulator can commit ex ante to an intervention policy that is a function of the price only. This last assumption is natural given that committing to a policy rule that is based on the publicly observed price may be feasible, while committing to a policy rule that is based on a privately observed signal is probably not. In view of the regulator's commitment, for this subsection only we drop the requirement that the time-consistency condition (3) must be satisfied in equilibrium.

The main thing to note about this case is that an equilibrium under regulatory commitment must entail fully revealing prices, i.e., in such an equilibrium every fundamental must be associated with a different price. This is because the regulator's intervention decision is now based only on the price. As a result, if two fundamentals had the same price, they would also have the same probability of intervention, and this would generate different prices. Thus, finding the optimal commitment policy boils down to finding the price function that maximizes the regulator's ex ante value function, subject to the constraint that the price function fully reveals the fundamentals.

The fact that the price function must be fully revealing implies that the regulator cannot achieve optimal intervention under commitment. This is because, as we saw in Figure 2, optimal intervention generates a price function that is not fully revealing – it has different fundamentals associated with the same price. The following proposition establishes a stronger result on the effectiveness of commitment. It says that, under commitment, the regulator will end up deviating from the benchmark full-information optimal intervention policy over a set of fundamentals that is at least of size $T\left(\hat{\theta}\right)$. Thus, commitment is not very effective in solving the problems raised in this paper.

Proposition 11 *If the regulator commits ex ante to an intervention policy based on the realization of the price of one security, he will not be able to achieve optimal intervention. The set of fundamentals at which the regulator deviates from the full-information optimal intervention policy is at least of size* $T\left(\hat{\theta}\right)$.

6 Other applications

The problem studied in this paper is one where an agent seeks information from a security price in order to decide whether to take an action that ultimately affects the value of the security. The distinctive feature of our problem is that the corrective action taken by the agent generates non-monotonicity in the price. That is, when fundamentals fall and push down the value of the security, the agent is expected to intervene and increase the value of the security. This non-monotonicity makes learning from the price about the fundamentals – and hence about the optimal decision – non-trivial. As discussed in the introduction, this non-monotonicity is ignored in the emerging literature on the feedback effect from asset prices to the real value of securities. In our writing, we focused on one concrete application where this problem arises: market-based bank regulation. Yet, the problem is much more general in scope and has many other applications. In this section, we describe two other important applications in financial economics.

The first application has to do with the decision of the board of directors to replace the

CEO of a firm. This decision has been discussed a lot in the empirical literature in financial economics. For example, see Warner, Watts, and Wruck (1988), Jenter and Kanaan (2006), and Kaplan and Minton (2006). These papers and others discuss the relation between the decision to fire a CEO and the stock price of a firm. At a theoretical level, this relation is parallel to the one in our model. In particular, using the notation in our model, let θ denote the expected cash flow of the firm, $T(\theta)$ denote the change in expected cash flow as a result of replacing the current CEO (which can be positive or negative), and C denote the cost that the board of directors has to bear when acting to replace the CEO. Suppose that the board of directors is benevolent, i.e., that it wishes to replace the CEO if and only if the benefit of doing this $T(\theta)$ is greater than the cost C. Finally, suppose that the benefit $T(\theta)$ is decreasing in θ. This is a natural assumption that reflects the idea that changing the CEO is beneficial only when the current CEO is not performing his job well.

As prior literature suggests, the decision of the board of directors will depend on the stock price of the firm. A low stock price can serve as an indicator that the CEO is not performing his job well, and thus that he should be replaced. As is clear from the description above, however, the problem that the board of directors faces in deciding whether or not to replace the CEO is very similar to the problem faced by the regulator in our model. In particular, since the stock price is forward looking it reflects not only expectations of cash flows generated by the current CEO but also expectations about whether the current CEO will be replaced by the board of directors. As a result, the link between the stock price and the decision to replace the CEO becomes non-trivial.

The second application deals with a large investor, who makes a decision to intervene in the firm's operations. This application is quite wide and can include situations such as an existing shareholder taking direct actions to affect managerial decisions (see the large literature on shareholder activism reviewed by Gillan and Starks (2007)), a debtholder (i.e., bank or bondholder) taking actions to affect managerial decisions (for example, because a covenant is triggered by changes in market value), or an outside investor who acquires shares in order to take over the firm and ultimately affect its value (e.g., Palepu (1986)). While the exact modelling will change from one situation to another, in general, these situations are very close to the problem studied in this paper. In particular, the investor taking the action is expected to get a benefit $V(\theta)$ that decreases in the fundamental θ (or, in other words, that increases in how much he can potentially fix with intervention), and pay a cost C. He can use the current price of the firm's share to gain information about θ, and thus about whether intervention is profitable or not. The problem, however, is that the share price reflects not only θ, but also the expectation regarding whether the investor will intervene or not. This complicates learning, just as in our model.

The two applications described here and the one which has been the focus of our paper have been the subject of many empirical papers. Our model has strong implications on how to conduct empirical analysis in these and other related settings. In particular, two key features of the model have to be taken into account. First, if agents (i.e., regulators, directors, investors) use the market price in their intervention decision, there will be dual causality between market prices and the intervention decision: market prices will reflect the regulator's action and affect it at the same time. In the context of shareholder activism in closed-end funds, Bradley, Brav, Goldstein, and Jiang (2006) conduct empirical analysis that indeed takes into account this dual causality. Second, when the information that agents have outside the financial market is not precise enough, our model generates equilibrium indeterminacy, which might make the relation between market prices and intervention more difficult to detect.

7 Conclusion

We study a rational expectations model of regulatory supervision of banks based on the market prices of bank securities. A key issue is that prices reflect both bank fundamentals and expectations of regulatory actions. In a wide range of cases, this generates non-monotonicity of the price with respect to fundamentals. When this happens, the regulator cannot easily extract information from the price to make an efficient intervention decision. We provide a complete characterization of the equilibrium outcomes of our model, and show that the ability of the regulator to extract information from the market depends on the gap between the regulator's and market's information quality. We also relate equilibrium outcomes to the type of security whose price the regulator observes. Convex securities lead to too much intervention, while concave securities lead to too little.

A key normative implication of our analysis is that market data and the regulator's own information should be treated as complements, in the sense that the regulator's own information is crucial for him in understanding whether shifts in market prices are due to changes in fundamentals or to changes in expectations regarding his own actions. We also provide implications for the potential efficacy of a number of distinct policy measures. Finally, we derive positive empirical implications on the relation between market prices and corrective actions that are based on them.

As we noted at the outset, the general insights from our analysis can be applied to many other settings in which private individuals use information from market prices to take actions that have a corrective effect on the value of the security. Examples include the decision of the board of directors on whether to replace a CEO, and the decision of various investors on whether to take actions to intervene in the operations of the firm.

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Mathematical appendix

We start with a couple of preliminaries. First, even though each combination of a fundamental and regulator signal has measure zero, to ease the exposition, we routinely evaluate the conditional probability $\Pr(\theta|\theta \in \{\theta_1, \theta_2\})$ as $1/2$ if $\theta \in \{\theta_1, \theta_2\}$ (recall that the fundamental θ is distributed uniformly). Likewise, we evaluate $\Pr(\theta|\theta \in \Theta) = 0$ if $\theta \notin \Theta$. We use parallel calculations for conditional expectations. The only proof in which it is important for us to proceed more formally in taking conditional probabilities and expectations is that of Proposition 3 (see the proof for the relevant details).

Second, it is convenient to state the following straightforward result separately.

Lemma 1 *Suppose that $T\left(\hat{\theta}\right) > 0$, and define $\breve{\theta} < \hat{\theta}$ by $\breve{\theta} + T\left(\breve{\theta}\right) = \hat{\theta}$. In any equilibrium,*

$$\Pr(I|\theta) = \begin{cases} 1 & \text{if } \theta < \max\left\{\hat{\theta} - 2\kappa, \hat{\theta} - T\left(\breve{\theta}\right)\right\} \\ 0 & \text{if } \theta > \min\left\{\hat{\theta} + 2\kappa, \hat{\theta} + T\left(\hat{\theta}\right)\right\} \end{cases}.$$

Proof of Lemma 1: Consider a fundamental $\theta < \hat{\theta} - 2\kappa$. At this fundamental, the regulator observes only signals below $\hat{\theta} - \kappa$. Such signals are never observed after any fundamental $\tilde{\theta} \geq \hat{\theta}$. As such, when the fundamental is θ the regulator knows that the fundamental lies to the left of $\hat{\theta}$. By time-consistency, he intervenes with probability 1. By a similar argument the regulator never intervenes if $\theta > \hat{\theta} + 2\kappa$.

Next, consider a fundamental $\theta < \hat{\theta} - T\left(\breve{\theta}\right) = \breve{\theta}$. In any equilibrium the price at θ is bounded above by $X(\theta) + U(\theta) = X(\theta + T(\theta)) < X(\breve{\theta} + T(\breve{\theta})) = X(\hat{\theta})$. Moreover, any fundamental $\tilde{\theta} \geq \hat{\theta}$ has a price that satisfies $P(\tilde{\theta}) \geq X(\tilde{\theta}) \geq X(\hat{\theta})$. Thus, in any equilibrium, if $\theta < \hat{\theta} - T\left(\breve{\theta}\right)$ then θ cannot share a price with *any* fundamental above $\hat{\theta}$. Again, by time-consistency the regulator intervenes with probability 1.

Finally, consider a fundamental $\theta > \hat{\theta} + T\left(\hat{\theta}\right)$. In any equilibrium the price at θ strictly exceeds $X\left(\hat{\theta} + T\left(\hat{\theta}\right)\right)$. Moreover, any fundamental $\tilde{\theta} \leq \hat{\theta}$ has a price that satisfies $P(\tilde{\theta}) \leq X(\tilde{\theta} + T\left(\tilde{\theta}\right)) \leq X\left(\hat{\theta} + T\left(\hat{\theta}\right)\right)$. Thus, in any equilibrium, if $\theta > \hat{\theta} + T\left(\hat{\theta}\right)$ then θ cannot share a price with *any* fundamental below $\hat{\theta}$. Again, by time-consistency the regulator intervenes with probability 0. ∎

Proof of Proposition 1: Existence is immediate. For uniqueness, suppose to the contrary that an equilibrium without optimal intervention exists. Any such equilibrium must feature a price P shared by a set of fundamentals Θ_P, where Θ_P has at least one element strictly less than $\hat{\theta}$ and at least one element strictly greater than $\hat{\theta}$. Fix any fundamental $\theta_2 \in \Theta_P$ that strictly exceeds $\hat{\theta}$. Let $q(\theta)$ denote the intervention probability at fundamental θ. Since all fundamentals in Θ_P share the same price, the following must hold for every $\theta \in \Theta_P$

$$q(\theta_2) X(\theta_2 + T(\theta_2)) + (1 - q(\theta_2)) X(\theta_2) - q(\theta) X(\theta + T(\theta)) - (1 - q(\theta)) X(\theta) = 0. \quad (7)$$

The left hand side of (7) can be rewritten as

$$\begin{aligned} & q(\theta_2)(X(\theta_2 + T(\theta_2)) - X(\theta + T(\theta))) \\ & + (1 - q(\theta_2)) X(\theta_2) - (q(\theta) - q(\theta_2)) X(\theta + T(\theta)) - (1 - q(\theta)) X(\theta). \end{aligned} \quad (8)$$

Note that for any $\theta \in \Theta_P \cap \left[\underline{\theta}, \hat{\theta}\right]$ the facts that $X(\theta + T(\theta))$ is increasing and $T\left(\hat{\theta}\right)$ is negative imply $X(\theta_2) > X\left(\hat{\theta}\right) \geq \max\{X(\theta + T(\theta)), X(\theta)\}$. If $q(\theta_2) = 0$ this delivers an immediate contradiction since $q(\theta) - q(\theta_2) \geq 0$ and so $(1 - q(\theta_2)) X(\theta_2) > (q(\theta) - q(\theta_2)) X(\theta + T(\theta)) + (1 - q(\theta)) X(\theta)$.

The remainder of the proof deals with the case in which $q(\theta_2) > 0$. Define $\theta^* = \sup \Theta_P \cap \left[\underline{\theta}, \hat{\theta}\right]$ and observe that for any $\theta \in \Theta_P \cap \left[\underline{\theta}, \hat{\theta}\right]$,

$$\begin{aligned} q(\theta) - q(\theta_2) &= \frac{1}{2\kappa}\left(\int_{\theta-\kappa}^{\theta+\kappa} I(P,\phi)\, d\phi - \int_{\theta_2-\kappa}^{\theta_2+\kappa} I(P,\phi)\, d\phi\right) \\ &= \frac{1}{2\kappa}\left(\int_{\theta-\kappa}^{\theta_2-\kappa} I(P,\phi)\, d\phi - \int_{\theta+\kappa}^{\theta^*+\kappa} I(P,\phi)\, d\phi - \int_{\theta^*+\kappa}^{\theta_2+\kappa} I(P,\phi)\, d\phi\right) \\ &= \frac{1}{2\kappa}\left(\int_{\theta-\kappa}^{\theta_2-\kappa} I(P,\phi)\, d\phi - \int_{\theta+\kappa}^{\theta^*+\kappa} I(P,\phi)\, d\phi\right), \end{aligned}$$

where the final equality follows since the price P and a signal above $\theta^* + \kappa$ together tell the regulator that the fundamental definitely exceeds $\hat{\theta}$. It follows that for any $\varepsilon > 0$ there exists some $\theta \in \Theta_P \cap \left[\underline{\theta}, \hat{\theta}\right]$ such that $q(\theta) - q(\theta_2) > -\varepsilon$. Hence for any $\varepsilon' > 0$ there exists some $\theta \in \Theta_P \cap \left[\underline{\theta}, \hat{\theta}\right]$ such that

$$(1 - q(\theta_2)) X(\theta_2) - (q(\theta) - q(\theta_2)) X(\theta + T(\theta)) - (1 - q(\theta)) X(\theta) > -\varepsilon'.$$

Finally, since $q(\theta_2)(X(\theta_2 + T(\theta_2)) - X(\theta + T(\theta))) > 0$ for $\theta \leq \hat{\theta}$, it is possible to choose $\theta \in \Theta_P \cap \left[\underline{\theta}, \hat{\theta}\right]$ such that (8) is strictly positive, contradicting (7) and completing the proof. ■

Proof of Proposition 2: In an optimal intervention equilibrium the regulator only intervenes for $\theta \leq \hat{\theta}$. What we need to check is that this policy is feasible. Under this intervention policy, $P(\theta) = X(\theta) + U(\theta)$ for $\theta \leq \hat{\theta}$, and $P(\theta) = X(\theta)$ for $\theta > \hat{\theta}$. As such, there are at most two fundamentals related to each price. For prices that are related to just one fundamental, the equilibrium price is trivially fully revealing. For prices that are related to two fundamentals, e.g., $\theta_1 < \hat{\theta} < \theta_2$,

$$X(\theta_1) + U(\theta_1) = X(\theta_2).$$

But from (1), this means that $X(\theta_1 + T(\theta_1)) = X(\theta_2)$, and thus $\theta_2 = \theta_1 + T(\theta_1)$. Since $T(\theta_1) \geq T\left(\hat{\theta}\right) > 2\kappa$, the regulator can distinguish between θ_1 and θ_2 with his own signal and follow his optimal intervention rule. ■

Proof of Proposition 3: The proof requires us to be more mathematically precise in our treatment of probabilities and expectations than is the case elsewhere in the paper. In particular, unlike elsewhere in the paper, we must assign conditional expectations and probabilities in cases where the conditioning set has infinitely many members yet is still null. Formally, let $\mathcal{B}$ denote the Borel algebra of $[\underline{\theta}, \bar{\theta}]$, so that $([\underline{\theta}, \bar{\theta}], \mathcal{B})$ is a measurable space. Let $\mu : \mathcal{B} \to [0,1]$ be the probability measure associated with the uniform distribution on $[\underline{\theta}, \bar{\theta}]$.

Let $\bar{\kappa} > 0$ be such that $X'(\theta) - \frac{U(\theta)}{2\kappa} < 0$ for all $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$, and fix an arbitrary $\kappa \in [0, \bar{\kappa}]$. We will show that in any equilibrium optimal intervention occurs almost surely. The proof is by contradiction: suppose to the contrary that there exists an equilibrium in which the regulator intervenes suboptimally over a non-null set of fundamentals. Clearly suboptimal intervention can only occur at non-revealing prices; and by Lemma 1, suboptimal intervention and non-revealing prices can only occur in $\left[\hat{\theta} - 2\kappa, \hat{\theta} + 2\kappa\right]$.

Throughout the proof we use the following definitions. Let $\mathcal{P}$ be the set of non-revealing prices. For each non-revealing price $P \in \mathcal{P}$ let Θ_P be the set of fundamentals associated with that price. Let $\Theta = \bigcup_{P \in \mathcal{P}} \Theta_P$ be the set of all fundamentals with a non-revealing price. By hypothesis, Θ has strictly positive measure.

Claim A: In an equilibrium in which the regulator intervenes suboptimally over a non-null set of fundamentals, $\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right]$ has strictly positive measure.

Proof of Claim A: Consider the conditional probability $\Pr\left(\Theta_P \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta_P\right)$. Clearly it equals $\Pr\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta_P\right)$. Moreover,

$$\int_{\theta \in \Theta} \Pr\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta_{P(\theta)}\right) \mu(d\theta) = \Pr\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta\right).$$

Suppose that contrary to the claim $\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right]$ is null. In this case, $\Pr\left(\Theta \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right] | \Theta_{P(\theta)}\right) = 0$ for almost all θ in Θ. But then the regulator would intervene optimally for almost all $\theta \in \Theta$: he would intervene with probability 1 at almost all $\theta \in \Theta$, since almost all members of Θ lie below $\hat{\theta}$. Since suboptimal intervention can potentially happen only at $\theta \in \Theta$, this contradicts an equilibrium in which the regulator intervenes suboptimally over a non-null set of fundamentals, and completes the proof of Claim A. ∎

For any signal realization ϕ, the regulator knows the true fundamental lies in the interval $[\phi - \kappa, \phi + \kappa]$. As such, for a price $P \in \mathcal{P}$ and signal ϕ the regulator's expected payoff (net of costs) from intervention is

$$v(P, \phi) \equiv E_\theta \left[V(\theta) - C | \theta \in \Theta_P \cap [\phi - \kappa, \phi + \kappa]\right]. \tag{9}$$

The heart of the proof lies in establishing:

Claim B: In an equilibrium of the kind described above, for any $P \in \mathcal{P}$: (1) $\sup \Theta_P \cap \left[\hat{\theta} - 2\kappa, \hat{\theta}\right] = \hat{\theta}$ and (2) $v(P, \phi = \theta + \kappa) \geq 0$ for any $\theta \in \Theta_P \cap \left[\hat{\theta} - 2\kappa, \hat{\theta}\right]$.

Proof of Claim B: Let θ_1 and $\theta_2 \in (\theta_1, \theta_1 + 2\kappa]$ be an arbitrary pair of members of Θ_P such that $\theta_1 \leq \hat{\theta}$ and $\theta_2 \geq \hat{\theta}$ (clearly all members of Θ_P cannot lie to the same side of $\hat{\theta}$, and at least one such pair must lie within 2κ of each other). Since θ_1 and θ_2 have the same price

$$X(\theta_1) + \frac{U(\theta_1)}{2\kappa} \int_{\theta_1 - \kappa}^{\theta_1 + \kappa} I(P, \phi)\, d\phi = X(\theta_2) + \frac{U(\theta_2)}{2\kappa} \int_{\theta_2 - \kappa}^{\theta_2 + \kappa} I(P, \phi)\, d\phi.$$

By assumption U is decreasing (X is concave and T is decreasing) and so $U(\theta_1) > U(\theta_2)$.

It follows that

$$X(\theta_2) \geq X(\theta_1) + \frac{U(\theta_1)}{2\kappa}\left(\int_{\theta_1-\kappa}^{\theta_1+\kappa} I(P,\phi)\,d\phi - \int_{\theta_2-\kappa}^{\theta_2+\kappa} I(P,\phi)\,d\phi\right).$$

Equivalently,

$$X(\theta_2) \geq X(\theta_1) + \frac{U(\theta_1)}{2\kappa}\left(\int_{\theta_1-\kappa}^{\theta_2-\kappa} I(P,\phi)\,d\phi - \int_{\theta_1+\kappa}^{\theta_2+\kappa} I(P,\phi)\,d\phi\right). \tag{10}$$

Define $\theta_1^* = \sup \Theta_P \cap \left[\hat{\theta} - 2\kappa, \hat{\theta}\right]$ and $\theta_2^* = \inf \Theta_P \cap \left[\hat{\theta}, \hat{\theta} + 2\kappa\right]$.

Suppose that either $v(P, \phi = \theta_1 + \kappa) < 0$ or $\theta_1^* < \hat{\theta}$. In the former case, $v(P,\phi) < 0$ for any signal ϕ above $\theta_1 + \kappa$ (since if $v(P,\phi)$ is strictly negative for some ϕ, the same is true for all higher ϕ). In the latter case, any signal ϕ above $\theta_1^* + \kappa$ rules out that $\theta \leq \hat{\theta}$. As such, by time-consistency $I(P,\phi) = 0$ for all $\phi > \theta_1 + \kappa$ in the former case, and $\phi > \theta_1^* + \kappa$ in the latter case. Since both sides of (10) are continuous in θ_1 and θ_2, it follows that

$$X(\theta_2) \geq X(\theta) + \frac{U(\theta)}{2\kappa}\int_{\theta-\kappa}^{\theta_2-\kappa} I(P,\phi)\,d\phi$$

for $\theta = \theta_1$ in the former case, and $\theta = \theta_1^*$ in the latter case. Certainly $I(P,\phi) = 1$ for all $\phi < \theta_2^* - \kappa$, since for these signal values the regulator knows that the fundamental lies to the left of $\hat{\theta}$. Thus the function Z defined by

$$Z(\theta, \theta_2) \equiv X(\theta_2) - X(\theta) - \frac{U(\theta)}{2\kappa}(\theta_2 - \theta)$$

is weakly positive at $(\theta, \theta_2) = (\theta_1, \theta_2^*)$ in the former case, and at (θ_1^*, θ_2^*) in the latter case. However, for any θ

$$\begin{aligned} Z(\theta,\theta) &= 0 \\ Z_2(\theta,\theta) &= X'(\theta) - \frac{U(\theta)}{2\kappa}. \end{aligned}$$

The function Z is concave in its second argument, and since $\kappa < \bar{\kappa}$, $Z_2(\theta,\theta) < 0$ for both $\theta = \theta_1$ and θ_1^*. So $Z(\theta, \theta_2)$ is strictly negative for all $\theta_2 > \theta$, where either $\theta = \theta_1$ or θ_1^*. This contradiction completes the proof of Claim B. ∎

We are now ready to complete the proof. By Claim B, for any $\varepsilon > 0$ and any $P \in \mathcal{P}$ there exists $\theta_{P,\varepsilon} \in \Theta_P \cap \left[\hat{\theta} - \varepsilon, \hat{\theta}\right]$ such that $v(P, \phi = \theta_{P,\varepsilon} + \kappa) \geq 0$. As such, the integral

$$\int_{\cup_{P\in\mathcal{P}}\left(\Theta_P \cap \left[\theta_{P,\varepsilon}, \theta_{P,\varepsilon}+2\kappa\right]\right)} v\left(P(\theta), \phi = \theta_{P(\theta),\varepsilon} + \kappa\right) \mu(d\theta) \tag{11}$$

is weakly positive. Since v is a conditional expectation (see its definition (9)), the integral is also equal to

$$\int_{\cup_{P\in\mathcal{P}}\left(\Theta_P\cap\left[\theta_{P,\varepsilon},\theta_{P,\varepsilon}+2\kappa\right]\right)} \left(V\left(\theta\right)-C\right)\mu\left(d\theta\right).$$

The domain of the integral (11) can be expanded as

$$\left(\Theta\cap\left[\hat{\theta},\hat{\theta}+2\kappa-\varepsilon\right]\right)\cup\bigcup_{P\in\mathcal{P}}\left(\Theta_P\cap\left[\theta_{P,\varepsilon},\hat{\theta}\right]\right)\cup\bigcup_{P\in\mathcal{P}}\left(\Theta_P\cap\left[\hat{\theta}+2\kappa-\varepsilon,\theta_{P,\varepsilon}+2\kappa\right]\right).$$

The term $V\left(\theta\right)-C$ is strictly negative over the first set above, with the single exception of at $\hat{\theta}$. For all ε small enough and by Claim A, the first set has strictly positive measure, while the other two have measures that approach zero. As such, the integral in expression (11) is strictly negative for ε small enough. The contradiction completes the proof. ∎

Proof of Proposition 4:

Let us start by characterizing the equilibria described in the proposition. There exist fundamentals $\theta_{12}>\theta_{02}>\hat{\theta}$ and a function $\theta_1^*:\left[\theta_{02},\theta_{12}\right]\rightarrow\left[\underline{\theta},\hat{\theta}\right]$ with $\theta_1^*\left(\theta_{12}\right)=\hat{\theta}$, such that for any set $Y_2\subset\left[\theta_{02},\theta_{12}\right]$ the following prices and intervention probabilities constitute an equilibrium:

1. [Optimal intervention above $\hat{\theta}$] If $\theta\geq\hat{\theta}$, the regulator intervenes with probability 0, and the price is $X\left(\theta\right)$.
2. [Under-intervention for some $\theta<\hat{\theta}$] If $\theta\in\theta_1^*\left(Y_2\right)$ the regulator intervenes with probability $\frac{\theta_1^{*-1}(\theta)-\theta}{2\kappa}>0$, and the price is $X\left(\theta\right)+\frac{\theta_1^{*-1}(\theta)-\theta}{2\kappa}U\left(\theta\right)$.
3. [Optimal intervention for some $\theta<\hat{\theta}$] If $\theta<\hat{\theta}$ and $\theta\notin\theta_1^*\left(Y_2\right)$, the regulator always intervenes, and the price is $X\left(\theta\right)+U\left(\theta\right)$.

To prove the existence of these equilibria, we start with the following lemma.

Lemma 2 *Suppose that X is concave. For $\kappa<T\left(\hat{\theta}\right)/2$ sufficiently close to $T\left(\hat{\theta}\right)/2$, there exists a unique $\theta_{12}>\hat{\theta}$ such that*

$$X\left(\hat{\theta}\right)+\frac{\theta_{12}-\hat{\theta}}{2\kappa}U\left(\hat{\theta}\right)=X\left(\theta_{12}\right).$$

Proof of Lemma 2: Define the function

$$Z(\theta_1, \theta_2) = X(\theta_1) + \frac{\theta_2 - \theta_1}{2\kappa} U(\theta_1) - X(\theta_2),$$

where $\theta_2 \geq \theta_1$. Intuitively, this is the difference between the price at fundamental θ_1 given an intervention probability $\frac{\theta_2 - \theta_1}{2\kappa}$, and the price at fundamental θ_2 given an intervention probability 0.

Observe that Z has the following properties:

$$\begin{aligned} Z_{22}(\theta_1, \theta_2) &> 0, \\ Z_{12}(\theta_1, \theta_2) &= \frac{U'(\theta_1)}{2\kappa} < 0, \\ Z(\theta, \theta) &= 0 \\ Z(\theta, \theta + 2\kappa) &= X(\theta + T(\theta)) - X(\theta + 2\kappa). \end{aligned}$$

Since $2\kappa < T\left(\hat{\theta}\right)$ we know that $Z\left(\hat{\theta}, \hat{\theta} + 2\kappa\right) > 0$ and $Z\left(\hat{\theta}, \hat{\theta}\right) = 0$. Since $Z_{22} > 0$, the result follows provided $Z_2\left(\hat{\theta}, \hat{\theta}\right) < 0$. We know that

$$\begin{aligned} Z_2\left(\hat{\theta}, \hat{\theta}\right) &= \frac{U\left(\hat{\theta}\right)}{2\kappa} - X'\left(\hat{\theta}\right) = \frac{X\left(\hat{\theta} + T\left(\hat{\theta}\right)\right) - X\left(\hat{\theta}\right)}{2\kappa} - X'\left(\hat{\theta}\right) \\ &= \frac{1}{2\kappa} \int_{\theta = \hat{\theta}}^{\hat{\theta} + T(\hat{\theta})} \left(X'(\theta) - \frac{2\kappa}{T\left(\hat{\theta}\right)} X'\left(\hat{\theta}\right) \right) d\theta. \end{aligned}$$

Since X is a concave function, $Z_2\left(\hat{\theta}, \hat{\theta}\right) < 0$ for all 2κ close enough to $T\left(\hat{\theta}\right)$. ∎

Now, define Z as in the proof of Lemma 2. Observe first that since $Z\left(\hat{\theta}, \theta_{12}\right) = Z\left(\hat{\theta}, \hat{\theta}\right) = 0$, and $Z_{22} > 0$, then $Z\left(\hat{\theta}, \theta_2\right) < 0$ for any $\theta_2 \in \left(\hat{\theta}, \theta_{12}\right)$. Moreover, $Z\left(\hat{\theta}, \theta_2\right)$ has a unique minimum, denoted as $\hat{\theta}_{02} \in \left(\hat{\theta}, \theta_{12}\right)$. Since for any $\theta_2 \in \left(\hat{\theta}_{02}, \theta_{12}\right)$, $Z\left(\hat{\theta}, \theta_2\right) < 0$ and $Z(\theta_2 - 2\kappa, \theta_2) > 0$, by continuity there exists some $\theta_1 < \hat{\theta}$, for which $Z(\theta_1, \theta_2) = 0$. We define a function, $\theta_1^*(\theta_2)$, where θ_1^* is the highest θ_1, below $\hat{\theta}$, for which $Z(\theta_1, \theta_2) = 0$. Economically, $\theta_1^*(\theta_2)$ is the fundamental which has the same market price as θ_2. We know that $\theta_1^*(\theta_{12}) = \hat{\theta}$.

We know $\theta_1^*(\theta_2)$ is a strictly increasing function. To see this, note that

$$Z(\theta_1, \theta_2) = Z\left(\hat{\theta}, \theta_2\right) - \int_{\theta_1}^{\hat{\theta}} Z_1(y, \theta_2)\, dy.$$

Since $Z\left(\hat{\theta},\theta_2\right)$ is increasing over the range $[\hat{\theta}_{02},\theta_{12}]$, and $Z_{12} < 0$, it follows that for any $\theta_1 \leq \hat{\theta}$, $Z(\theta_1,\theta_2)$ is increasing in θ_2 over $[\hat{\theta}_{02},\theta_{12}]$. Thus, the highest θ_1 at which $Z(\theta_1,\theta_2) = 0$ is strictly increasing in θ_2, implying that $\theta_1^*(\theta_2)$ is a strictly increasing function.

Since $\theta_1^*(\theta_{12}) = \hat{\theta}$, the function $\frac{V(\theta_2)+V(\theta_1^*(\theta_2))-2C}{2}$ is strictly negative at $\theta_2 = \theta_{12}$. Define θ_{02} as the maximum of $\hat{\theta}_{02}$ and the supremum value such that $\frac{V(\theta_2)+V(\theta_1^*(\theta_2))-2C}{2} \geq 0$. As such, $\theta_1^*(\cdot)$ is increasing and $\frac{V(\theta_2)+V(\theta_1^*(\theta_2))-2C}{2} < 0$ over the interval $[\theta_{02},\theta_{12}]$.

We have now defined the values θ_{02} and θ_{12} that were used to characterize the equilibria in the beginning of the proof. It remains to show that the prices and intervention probabilities, as described above, indeed form equilibria. This requires showing that the prices are rational given the intervention probabilities, and that the intervention probabilities result from the regulator's optimal behavior given the information in the price and his own signal. It is immediate to show that the prices in the proposition statement are rational given the corresponding intervention probabilities. Thus, we turn to show that the intervention probabilities result from the regulator's optimal behavior. We will do this by analyzing different ranges of the fundamentals separately.

For a fundamental $\theta \geq \hat{\theta}$ and $\theta \notin Y_2$, the price is $X(\theta)$. The same price may be observed at the fundamental $\theta' = \theta - T(\theta') < \hat{\theta}$. Since $2\kappa < T\left(\hat{\theta}\right) \leq T(\theta')$, the regulator's signal will indicate for sure that the fundamental is θ and not θ'. Hence, the regulator will optimally choose not to intervene, generating intervention probability of 0, as is stated in the proposition. Note that the same price cannot be observed at any fundamental above θ'. Observing such a price at a fundamental above θ' would imply that the fundamental belongs to the set $\theta_1^*(Y_2)$, but this contradicts the fact that $\theta \notin Y_2$.

For a fundamental $\theta \geq \hat{\theta}$ and $\theta \in Y_2$, the price is again $X(\theta)$. As before, the same price may be observed at the fundamental $\theta' = \theta - T(\theta')$ without having an effect on the decision of the regulator not to intervene at θ. Here, however, the same price will also be observed at the fundamental $\theta_1^*(\theta)$. This is because the fundamental $\theta_1^*(\theta) \in \theta_1^*(Y_2)$ generates a price of $X(\theta_1^*(\theta)) + \frac{\theta-\theta_1^*(\theta)}{2\kappa}U(\theta_1^*(\theta))$, which by construction is equal to $X(\theta)$. (Note that the same price will not be observed at any other fundamental in the set $\theta_1^*(Y_2)$, since $X(\theta)$

and $\theta_1^*(\theta)$ are strictly increasing in θ.) Thus, at the fundamental θ, the regulator observes a price that is consistent with both θ and $\theta_1^*(\theta)$, and may observe a signal that is also consistent with both of them. If this happens, given the uniform distribution of noise in the regulator's signal, the regulator will not intervene as long as $\frac{V(\theta)+V(\theta_1^*(\theta))-2C}{2} < 0$. By construction, this is true for all $\theta \in Y_2$, and thus, at the fundamental θ, the regulator will intervene with probability 0, as is stated in the proposition.

For a fundamental $\theta < \hat{\theta}$ and $\theta \notin \theta_1^*(Y_2)$, the price is $X(\theta) + U(\theta)$. The same price may be observed at the fundamental $\theta + T(\theta) \geq \hat{\theta}$ and also at some $\theta' < \hat{\theta}$ in $\theta_1^*(Y_2)$. Since $2\kappa < T\left(\hat{\theta}\right) \leq T(\theta)$, the regulator's signal at the fundamental θ will indicate for sure that the fundamental is not $\theta + T(\theta)$. Hence, the regulator will know that the fundamental is below $\hat{\theta}$, and will optimally choose to intervene, generating intervention probability of 1, as is stated in the proposition.

Finally, for a fundamental $\theta < \hat{\theta}$ and $\theta \in \theta_1^*(Y_2)$, the price is $X(\theta) + \frac{\theta_1^{*-1}(\theta)-\theta}{2\kappa} U(\theta)$. As follows from the arguments above, the same price will be observed at the fundamental $\theta_1^{*-1}(\theta)$, and also may be observed at some fundamental $\theta' < \hat{\theta}$, where $\theta' \notin \theta_1^*(Y_2)$. (As argued before, two fundamentals in the set $\theta_1^*(Y_2)$ cannot have the same price.) As also follows from the arguments above, the regulator will optimally choose not to intervene if and only if his signal is consistent with both θ and $\theta_1^{*-1}(\theta)$ (the signal cannot be consistent with both $\theta_1^{*-1}(\theta)$ and θ'). Due to the uniform distribution of noise in the regulator's signal, this generates an intervention probability of $\frac{\theta_1^{*-1}(\theta)-\theta}{2\kappa}$, as is stated in the proposition. ∎

Proof of Proposition 5: Suppose to the contrary that there exists an equilibrium without optimal intervention and in which the probability of intervention for all $\theta < \hat{\theta}$ is 1. In this equilibrium there must exist some $\theta_2 > \hat{\theta}$ such that $E[I|\theta_2] > 0$. Because $\theta_2 > \hat{\theta}$, it follows that there must exist $\theta_1 \in [\theta_2 - 2\kappa, \hat{\theta})$ with the same price as θ_2. Moreover, because $E[I|\theta] = 1$ for all $\theta < \hat{\theta}$, the fundamental θ_1 is the unique fundamental to the left of $\hat{\theta}$ with the same price as θ_2. So the intervention policy I in this equilibrium must satisfy

$$I(P(\theta_2), \phi) = \begin{cases} 1 & \text{if } \phi \in (\theta_1 - \kappa, \theta_1 + \kappa) \\ 0 & \text{if } \phi \in (\theta_2 - \kappa, \theta_2 + \kappa) \text{ and } \phi \notin (\theta_1 - \kappa, \theta_1 + \kappa) \end{cases}.$$

As such, the expected intervention probability at θ_2 is

$$E[I|\theta_2] = \Pr((\theta_2 + \xi) \in (\theta_2 - \kappa, \theta_1 + \kappa)) = 1 - \frac{\theta_2 - \theta_1}{2\kappa}.$$

The fact that the price at θ_1 is equal to the price at θ_2 implies that

$$X(\theta_1 + T(\theta_1)) = \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) X(\theta_2 + T(\theta_2)) + \frac{\theta_2 - \theta_1}{2\kappa} X(\theta_2).$$

Since $X(\cdot)$ is a concave function, this means that

$$\theta_1 + T(\theta_1) < \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right)(\theta_2 + T(\theta_2)) + \frac{\theta_2 - \theta_1}{2\kappa}\theta_2.$$

Rearranging the terms, we get

$$T(\theta_2)\left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) + \theta_2 > \theta_1 + T(\theta_1).$$

However, we know that $T(\theta_2) \leq T(\theta_1)$, and so

$$\begin{aligned} T(\theta_2)\left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) + \theta_2 &\leq T(\theta_1)\left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) + \theta_2 \\ &\leq \theta_1 + T(\theta_1) - (\theta_2 - \theta_1)\left(\frac{T(\theta_1)}{2\kappa} - 1\right) \\ &< \theta_1 + T(\theta_1) \end{aligned}$$

since $2\kappa < T\left(\hat{\theta}\right) \leq T(\theta_1)$. The resulting contradiction completes the proof. ■

Proof of Proposition 6: Suppose to the contrary that an equilibrium exists. Let $P(\cdot)$ be the equilibrium price function. We know that there cannot be a fully-revealing equilibrium (see the main text immediately prior to the proposition statement). Define Θ^* to be the non-empty set of fundamentals at which the price is not fully-revealing, i.e.,

$$\Theta^* = \left\{\theta : \exists \theta' \neq \theta \text{ such that } P(\theta) = P(\theta')\right\}.$$

Given Θ^*, define $\theta^* = \sup \Theta^*$. We prove the following claims.

Claim 1: If $T\left(\hat{\theta}\right) > 0$ then $T(\theta) \geq 0$ for every $\theta \leq \theta^*$.

Proof of Claim 1: Define θ_{T0} as the minimal θ such that $T(\theta) = 0$. (If $T\left(\bar{\theta}\right) > 0$ then the claim holds vacuously.) Consider any fundamental $\theta \geq \theta_{T0}$. The price exceeds $X(\theta + T(\theta)) \geq X(\theta_{T0} + T(\theta_{T0})) = X(\theta_{T0})$. On the other hand, if $\theta' < \theta_{T0}$ the price is bounded above by $X(\theta' + T(\theta')) < X(\theta_{T0} + T(\theta_{T0})) = X(\theta_{T0})$. So it is impossible for the fundamental θ to share the same price as any fundamental θ' below θ_{T0}. Note that $\theta_{T0} > \hat{\theta}$ since $T\left(\hat{\theta}\right) > 0$. So the price of θ is consistent only with fundamentals above θ_{T0}, which in turn exceeds $\hat{\theta}$. It follows that the regulator never intervenes above θ_{T0}, implying that prices in this region are fully revealing, and so $\theta_{T0} \geq \theta^*$.

Claim 2: If $\theta > \max\left\{\theta^*, \hat{\theta}\right\}$ then $P(\theta) = X(\theta)$; and if $\theta \leq \max\left\{\theta^*, \hat{\theta}\right\}$ then $P(\theta) \leq X\left(\max\left\{\theta^*, \hat{\theta}\right\}\right)$.

Proof of Claim 2: By definition, if $\theta > \theta^*$ the price is fully-revealing. So if $\theta > \hat{\theta}$ also, the regulator does not intervene, and $P(\theta) = X(\theta)$. So for any $\theta \in \left(\max\left\{\theta^*, \hat{\theta}\right\}, \infty\right)$, the price is $X(\theta)$. Next, suppose that contrary to the claim $P(\theta') > X\left(\max\left\{\theta^*, \hat{\theta}\right\}\right)$ for some $\theta' \leq \max\left\{\theta^*, \hat{\theta}\right\}$. But then there exists $\theta > \max\left\{\theta^*, \hat{\theta}\right\} \geq \theta^*$ such that $P(\theta) = P(\theta')$, contradicting the fact that $\theta^* = \sup \Theta^*$. This completes the proof of Claim 2.

Claim 3: $\theta^* > \hat{\theta}$.

Proof of Claim 3: Suppose to the contrary that $\theta^* \leq \hat{\theta}$, so that $\max\left\{\theta^*, \hat{\theta}\right\} = \hat{\theta}$. By Claim 2, $P(\theta) = X(\theta)$ if $\theta > \hat{\theta}$, and $P(\theta) \leq X\left(\hat{\theta}\right)$ for $\theta \leq \hat{\theta}$. As such, whenever the true fundamental is strictly below $\hat{\theta}$ the regulator knows either that the fundamental is strictly below $\hat{\theta}$; or that the fundamental is either strictly below $\hat{\theta}$ or equal to $\hat{\theta}$, with a positive probability of both. So the regulator intervenes with probability one for any $\theta < \hat{\theta}$. But then the price is not below $X\left(\hat{\theta}\right)$ for any θ close to $\hat{\theta}$. This contradiction completes the proof of the Claim 3.

Claim 4: $P(\theta^*) = X(\theta^*)$, and so $E(I|\theta^*) = 0$.

Proof of Claim 4: From Claims 2 and 3, $P(\theta) \leq X(\theta^*)$ for $\theta \leq \theta^*$. The claim follows since by Claim 1 $P(\theta^*) \geq X(\theta^*)$.

Now, consider first the case where $\theta^* \in \Theta^*$. By construction, there exists a fundamental $\theta' < \theta^*$ such that: $P(\theta') = X(\theta') + E(I|\theta') U(\theta') = X(\theta^*)$. Observe that $\theta^* < \theta' + 2\kappa$, since if $\theta' \leq \theta^* - 2\kappa$ the price at θ' is at most $X(\theta^* - 2\kappa + T(\theta^* - 2\kappa))$, which since $2\kappa > T\left(\hat{\theta} - 2\kappa\right) \geq T(\theta^* - 2\kappa)$ is less than $P(\theta^*) = X(\theta^*)$.

Since $E(I|\theta^*) = 0$, the regulator does not intervene at signals above $\theta^* - \kappa$. Thus,

$E(I|\theta') \leq \Pr(\theta' + \xi \leq \theta^* - \kappa) = \frac{\theta^* - \theta'}{2\kappa}$. Define the function $Z(\theta', \theta^*)$ as follows:

$$Z(\theta', \theta^*) \equiv X(\theta') + U(\theta') \frac{\theta^* - \theta'}{2\kappa} - X(\theta^*).$$

By the above arguments, in the proposed equilibrium, $Z(\theta', \theta^*) \geq 0$. We know that $Z(\theta', \theta') = 0$, and (since $2\kappa > T\left(\hat{\theta} - 2\kappa\right) \geq T(\theta')$) that $Z(\theta', \theta' + 2\kappa) = X(\theta' + T(\theta')) - X(\theta' + 2\kappa) < 0$. Since the security is concave, $Z_{22} > 0$. Thus, there are no θ' and $\theta^* \in (\theta', \theta' + 2\kappa)$ for which $Z(\theta', \theta^*) \geq 0$. This is a contradiction to the proposed equilibrium.

Suppose now that $\theta^* \notin \Theta^*$. There exists some sequence $(\theta_i)_{i=0}^{\infty} \subset \Theta^*$ that converges to θ^*. Moreover, by Claims 1 - 4, $E(I|\theta_i) \to 0$ as $i \to \infty$: for if this is not true, there is a $\theta_i \leq \theta^*$ at which the price is above $X(\theta^*)$. For each θ_i in this sequence there exists at least one fundamental, θ'_i, at which the price is the same and which lies to the left of $\hat{\theta}$. (If instead all fundamentals with price $P(\theta_i)$ were to the right of $\hat{\theta}$, no intervention would occur, and they could not have the same price.) So $X(\theta'_i) + E(I|\theta'_i) U(\theta') = X(\theta_i) + E(I|\theta_i) U(\theta_i)$. Note that $\theta_i - \theta'_i$ is bounded away from 0 as $i \to \infty$ since $\theta_i \to \theta^* > \hat{\theta}$. We know that

$$\begin{aligned} E(I|\theta'_i) &= \int_{\theta'_i - \kappa}^{\theta'_i + \kappa} I(P(\theta_i), \phi) \frac{1}{2\kappa} d\phi \\ &\leq \int_{\theta'_i - \kappa}^{\theta_i - \kappa} I(P(\theta_i), \phi) \frac{1}{2\kappa} d\phi + \int_{\theta_i - \kappa}^{\theta_i + \kappa} I(P(\theta_i), \phi) \frac{1}{2\kappa} d\phi \\ &\leq \frac{\theta_i - \theta'_i}{2\kappa} + E(I|\theta_i). \end{aligned}$$

Define

$$\begin{aligned} \varepsilon_i &\equiv E(I|\theta_i) \left(U(\theta'_i) - U(\theta_i)\right) \\ \hat{Z}(\theta'_i, \theta_i) &\equiv X(\theta'_i) + U(\theta'_i) \frac{\theta_i - \theta'_i}{2\kappa} - X(\theta_i) \\ Z(\theta'_i, \theta_i) &\equiv \hat{Z}(\theta'_i, \theta_i) + \varepsilon_i \end{aligned}$$

By the above arguments, in the proposed equilibrium, $Z(\theta'_i, \theta_i) \geq 0$. We know that ε_i approaches 0 (the value of intervention, $U(\theta)$, is bounded above by the maximum value of T). We know that $\hat{Z}(\theta'_i, \theta'_i) = 0$, and that $\hat{Z}(\theta'_i, \theta'_i + 2\kappa) = X(\theta'_i + T(\theta'_i)) - X(\theta'_i + 2\kappa) < 0$ for all i large enough (since $2\kappa > T\left(\hat{\theta} - 2\kappa\right) \geq T(\theta'_i)$). Since the security is concave, $\hat{Z}_{22} > 0$.

Thus, for any θ_i between θ_i' and $\theta_i' + 2\kappa$, $Z\left(\theta_i', \theta_i\right) \leq \varepsilon_i + \frac{(\theta_i - \theta_i')(X(\theta_i' + T(\theta_i')) - X(\theta_i' + 2\kappa))}{2\kappa}$. This implies that $Z\left(\theta_i', \theta_i\right) \geq 0$ can hold only if $\theta_i' \leq \theta_i \leq \theta_i' + \frac{2\kappa\varepsilon_i}{X(\theta_i' + 2\kappa) - X(\theta_i' + T(\theta_i'))}$. Then, since ε_i approaches 0, there are no θ_i' and θ_i that are bounded away from each other for which $Z\left(\theta_i', \theta_i\right) \geq 0$. This is a contradiction to the proposed equilibrium. ∎

Proof of Proposition 7: Part (A). The first half is immediate. For the second half, it suffices to show that in any equilibrium of the trading game with prices posted in all states the mapping from fundamentals to prices satisfies the rational expectations equilibrium condition (2). To see this, note that since speculators have the same information as the market maker, if the posted price is not equal to the security's expected payoff then speculators could buy (or sell) the security to make positive profits. In this case, the market maker would make negative profits.

Part (B). To complete the description of the equilibrium, let the regulator's off-equilibrium-path beliefs be such that if he observes a signal ϕ and a price corresponding in equilibrium to fundamental $\theta < \phi - \kappa$ (respectively, $\theta > \phi + \kappa$), then he believes the fundamental is $\phi - \kappa$ (respectively, $\phi + \kappa$). Moreover, the regulator's intervention decision at fundamental $\theta \in (\tilde{\theta}, \tilde{\theta} + T\left(\tilde{\theta}\right)]$ and signal ϕ is determined by the sign of

$$E\left[V\left(\theta'\right)|\theta' \in (\tilde{\theta}, \tilde{\theta} + T\left(\tilde{\theta}\right)] \cap [\phi - \kappa, \phi + \kappa]\right].$$

In the conjectured equilibrium, whenever a price is posted it perfectly reveals the fundamental. So by construction, the regulator's intervention decision is a best response. It remains only to check that the market maker has no profitable deviation.

For use below, note that by construction $\tilde{\theta} \leq \hat{\theta} \leq \tilde{\theta} + T\left(\tilde{\theta}\right)$; and by assumption $\hat{\theta} - 2\kappa + T\left(\hat{\theta} - 2\kappa\right) < \hat{\theta} = \check{\theta} + T\left(\check{\theta}\right)$, implying $\hat{\theta} - 2\kappa < \check{\theta}$ and hence $2\kappa > T\left(\hat{\theta} - 2\kappa\right) \geq T\left(\theta\right)$ for all $\theta \geq \check{\theta}$.

Consider a realization of the fundamental $\theta \leq \tilde{\theta}$. For these fundamentals the market maker posts a price and makes zero profits. He cannot profit by not posting a price. If he posts a higher price $p > X\left(\theta + T\left(\theta\right)\right)$ then regardless of the regulator's response the value of the security is less than p, and so speculators will short the security and the market maker will lose money. If he posts a lower price $p < X\left(\theta + T\left(\theta\right)\right)$ then (given the beliefs specified) the regulator will intervene, implying that the value of the security exceeds p and the market maker will lose money. By a similar argument, the market maker does not have a profitable deviation if $\theta > \tilde{\theta} + T\left(\tilde{\theta}\right)$.

Next, suppose $\theta \in (\tilde{\theta}, \tilde{\theta} + T\left(\tilde{\theta}\right)]$, the no price region. First, consider a deviation by the market maker in which he posts a price $p > X\left(\tilde{\theta} + T\left(\tilde{\theta}\right)\right)$. Let $\theta' > \tilde{\theta} + T\left(\tilde{\theta}\right) \geq \theta$ be such that $X\left(\theta'\right) = p$. Whenever the regulator observes $\phi \in \left[\theta' - \kappa, \theta + \kappa\right]$ he believes the fundamental is θ' and does not intervene. So the intervention probability is bounded above by $\frac{\theta'-\theta}{2\kappa}$, and so the security value is bounded above by

$$\frac{\theta' - \theta}{2\kappa} X\left(\theta + T\left(\theta\right)\right) + \left(1 - \frac{\theta' - \theta}{2\kappa}\right) X\left(\theta\right).$$

This is strictly less than the quoted price $X\left(\theta'\right)$ for all $\theta' \in (\theta, \theta + 2\kappa]$, since X is concave and $2\kappa > T\left(\theta\right)$. Likewise, if $\theta' > \theta + 2\kappa$ then $X\left(\theta'\right) > X\left(\theta + 2\kappa\right) \geq X\left(\theta + T\left(\theta\right)\right)$, and so again the quoted price must exceed the value of security. So the regulator loses money from a deviation of this form.

Second, consider a deviation by the market maker in which he posts a price $p \leq X\left(\tilde{\theta} + T\left(\tilde{\theta}\right)\right)$. Let $\theta' \leq \tilde{\theta}$ be such that $X\left(\theta' + T\left(\theta'\right)\right) = p$. So the regulator believes the fundamental is θ' if $\phi \in \left[\theta - \kappa, \theta' + \kappa\right]$, and $\phi - \kappa$ if $\phi \in (\theta' + \kappa, \theta + \kappa]$. Since $\theta' \leq \tilde{\theta} \leq \hat{\theta}$, it follows that the regulator intervenes with probability 1 if $\theta < \hat{\theta}$, and with probability $\frac{\hat{\theta}+\kappa-(\theta-\kappa)}{2\kappa} = 1 - \frac{\theta-\hat{\theta}}{2\kappa}$ if $\theta \geq \hat{\theta}$. The value of the security under this deviation is thus $X\left(\theta + T\left(\theta\right)\right)$ if $\theta < \hat{\theta}$, and

$$\left(1 - \frac{\theta - \hat{\theta}}{2\kappa}\right) X\left(\theta + T\left(\theta\right)\right) + \frac{\theta - \hat{\theta}}{2\kappa} X\left(\theta\right)$$

if $\theta \geq \hat{\theta}$. In the former case the value of the security certainly lies strictly above the quoted price of $X\left(\theta' + T\left(\theta'\right)\right)$, causing the market maker to lose money from this deviation. The same is true for the latter case for $\theta \leq \check{\theta} + T\left(\check{\theta}\right) = \hat{\theta}$ and $\theta' \leq \check{\theta}$. Finally, by continuity, this is also the case for $\theta \leq \tilde{\theta} + T\left(\tilde{\theta}\right)$ and $\theta' \leq \tilde{\theta}$ for all $\tilde{\theta}$ sufficiently close to $\check{\theta}$.

Finally, note that (except for when $\tilde{\theta} = \check{\theta}$ or $\hat{\theta}$) the equilibrium entails suboptimal intervention. To see this, fix an equilibrium, and consider the regulator's action when he sees a signal $\phi = \hat{\theta}$ and no price. If he intervenes, this implies that with positive probability he suboptimally intervenes for some $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$ to the right of $\hat{\theta}$. Likewise, if the regulator does not intervene, then this implies that with positive probability he suboptimally does not intervene for some $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$ to the left of $\hat{\theta}$. ∎

Proof of Proposition 8: Suppose the regulator observes the price of securities A and B, where security A is strictly convex and security B is strictly concave. The heart of the proof is the following straightforward claim:

Claim: For any pair of fundamentals θ_1 and $\theta_2 \neq \theta_1$ there is no probability $q \in (0,1)$ such that

$$X_s(\theta_1) + qU_s(\theta_1) = X_s(\theta_2) \text{ for securities } s = A, B \tag{12}$$

or

$$X_s(\theta_1) + qU_s(\theta_1) = X_s(\theta_2 + T(\theta_2)) \text{ for securities } s = A, B. \tag{13}$$

Proof of Claim: Observe that

$$X_s(\theta_1) + qU_s(\theta_1) = (1-q)X_s(\theta_1) + qX_s(\theta_1 + T(\theta_1)) \left\{ \begin{array}{c} > \\ < \end{array} \right\} X_s(\theta_1 + qT(\theta_1)) \text{ if security } s \text{ is} \left\{ \begin{array}{c} \text{convex} \\ \text{concave} \end{array} \right\}$$

Since X_s is monotone strictly increasing for both securities, it is immediate that neither (12) nor (13) can hold. ∎

The proof of the main result applies this Claim. Consider any equilibrium, and let Θ be the set of fundamentals that share the same price vector as a fundamental at which intervention is suboptimal. Suppose that (contrary to the claimed result) the set Θ is non-empty. Let θ^* be its supremum. Clearly if $\theta^* \leq \hat{\theta}$ then for all equilibrium prices associated with fundamentals Θ the regulator would know the true fundamental lies below $\hat{\theta}$, and would intervene optimally. So $\theta^* > \hat{\theta}$. Moreover, by Lemma 1, $\theta^* \leq \hat{\theta} + 2\kappa < \hat{\theta} + T(\hat{\theta})$. For use below, let θ^{**} be such that $\theta^{**} + T(\theta^{**}) = \theta^*$. Note that $\theta^{**} \leq \hat{\theta}$, since otherwise θ^* cannot be the supremum of Θ. So $T(\theta^{**}) \geq T(\hat{\theta})$.

By construction, for fundamentals $\theta > \theta^*$ the regulator intervenes optimally, so $P(\theta) = X(\theta)$. Therefore, for all fundamentals $\theta \in \Theta$ the equilibrium price vector satisfies $P(\theta) \leq X(\theta^*)$. Consider an arbitrary sequence $\{\theta_i\} \subset \Theta$ such that $\theta_i \to \theta^*$. The intervention probabilities converge to zero along this sequence, $E[I|\theta_i] \to 0$ (otherwise, the equilibrium price would strictly exceed $X(\theta^*)$ for some θ_i). There are two cases to consider:

Case A: On the one hand, suppose there exists some $\varepsilon > 0$ and some infinite subsequence $\{\theta_j\} \subset \{\theta_i\}$ such that for each θ_j there is a fundamental $\theta'_j \neq \theta_j$ with the same price, and $E\left[I|\theta'_j\right] \in [\varepsilon, 1-\varepsilon]$. It follows that there is a subsequence $\{\theta_k\} \subset \{\theta_j\}$ such that for each θ_k there is a fundamental $\theta'_k \neq \theta_k$ with the same price, and $E\left[I|\theta'_k\right]$ converges to $q \in [\varepsilon, 1-\varepsilon]$ as $k \to \infty$. Since for all k

$$X_s(\theta_k) + E[I|\theta_k] U_s(\theta_k) = X_s(\theta'_k) + E[I|\theta'_k] U_s(\theta'_k)$$

for securities $s = A$, B, and the left-hand side converges to $X_s(\theta^*)$, it follows that $\{\theta'_k\}$ must converge also, to θ' say. Thus $X_s(\theta^*) = X_s(\theta') + qU_s(\theta')$ for securities $s = A, B$, directly contradicting the above Claim.

Case B: On the other hand, suppose that Case A does not hold. So there exists an infinite subsequence $\{\theta_j\} \subset \{\theta_i\}$ such that for each fundamental θ'_j possessing the same price as θ_j the intervention probability $E\left[I|\theta'_j\right]$ is either less than $1/j$ or greater than $1 - 1/j$. It follows that for j large, all fundamentals with the same price vector as θ_j are close to either θ^* (if the intervention probability is close to 0) or $\theta^* - T(\theta^{**})$ (if the intervention probability is close to 1): formally, there exists some sequence ε_j such that $\varepsilon_j \to 0$ and such that $\theta'_j \in [\theta^* - T(\theta^{**}) - \varepsilon_j, \theta^* - T(\theta^{**}) + \varepsilon_j] \cup [\theta^* - \varepsilon_j, \theta^*]$. But for j large enough, $\theta^* - \varepsilon_j > \hat{\theta}$, $\theta^* - T(\theta^{**}) + \varepsilon_j < \hat{\theta}$, and $(\theta^* - \varepsilon_j) - (\theta^* - T(\theta^{**}) + \varepsilon_j) = T(\theta^{**}) - 2\varepsilon_j > 2\kappa$. That is, for j large, if the regulator observes price vector $P(\theta_j)$ and his own signal, it knows with certainty which side of $\hat{\theta}$ the fundamental lies. As such, he intervenes optimally, giving a contradiction. ∎

Proof of Proposition 9: Exactly as in Proposition 6 a fully-revealing equilibrium cannot exist. Suppose a non-fully revealing equilibrium exists. So at some set of fundamentals Θ^* the prices of both the concave and convex securities must be the same for at least two distinct fundamentals. That is, the set

$$\Theta^* \equiv \left\{\theta : \exists \theta' \neq \theta \text{ such that } P_i(\theta) = P_i(\theta') \text{ for all securities } i\right\}$$

is non-empty. The proof of Proposition 6 applies, and gives a contradiction. ∎

Proof of Proposition 10: First, in any equilibrium where there exist $\theta_1 < \theta_2$ with the same debt security price, the expected intervention probabilities $E[\theta_1|I]$ and $E[\theta_2|I]$ must differ (otherwise prices would not be identical). Given that the probability of intervention can be directly inferred from $Q(\theta)$, then the regulator can always infer θ based on $P(\theta)$ and $Q(\theta)$. Then, the regulator will choose to intervene when $\theta \leq \hat{\theta}$, and not intervene otherwise. The same is true if the equilibrium prices of the standard security are fully revealing. Thus, if there is an equilibrium, it must feature optimal intervention.

Second, we show that optimal intervention is indeed an equilibrium. In such an equilibrium, the price of the debt security is $X(\theta + T(\theta))$ for $\theta < \hat{\theta}$ and $X(\theta)$ for $\theta > \hat{\theta}$. The regulator security has a price of 1 for $\theta < \hat{\theta}$ and 0 for $\theta > \hat{\theta}$. Then, independent of the regulator's signal, the regulator chooses to intervene below $\hat{\theta}$ and not intervene above $\hat{\theta}$. This is indeed consistent with the prices, so optimal intervention is an equilibrium. ■

Proof of Proposition 11: Denote the size of the set of parameters in $\left[\hat{\theta} - T\left(\check{\theta}\right), \hat{\theta}\right]$ over which the regulator intervenes optimally as λ^- (where $\check{\theta}$ is as defined in Lemma 1), and the size of the set of parameters in $\left[\hat{\theta}, \hat{\theta} + T\left(\hat{\theta}\right)\right]$ over which the regulator intervenes optimally as λ^+.

By the shape of the price function under optimal intervention (see Figure 2), every fundamental $\theta \in \left[\hat{\theta} - T\left(\check{\theta}\right), \hat{\theta}\right]$ that exhibits optimal intervention implies that the intervention decision at $\theta + T(\theta) \in \left[\hat{\theta}, \hat{\theta} + T\left(\hat{\theta}\right)\right]$ is suboptimal. This is because optimal intervention at both θ and $\theta + T(\theta)$ implies that the two fundamentals have the same price, but this is impossible in a commitment equilibrium. Thus, the set of fundamentals with optimal intervention in $\left[\hat{\theta} - T\left(\check{\theta}\right), \hat{\theta}\right]$ cannot be greater than the set of fundamentals with suboptimal intervention in $\left[\hat{\theta}, \hat{\theta} + T\left(\hat{\theta}\right)\right]$. That is, $\lambda^- \leq T\left(\hat{\theta}\right) - \lambda^+$, which implies that $\lambda^- + \lambda^+ \leq T\left(\hat{\theta}\right)$. This completes the proof. ■