When Two and Two is Not Equal to Four:

Errors in Processing Multiple Percentage Changes

HAIPENG (ALLAN) CHEN

AKSHAY R. RAO*

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* Haipeng (Allan) Chen is Assistant Professor, Marketing Department, 505 Kosar/Epstein Building, University of Miami, Coral Gables, FL 33124 (hchen@miami.edu). Akshay R. Rao is General Mills Professor of Marketing and Director, Institute for Research in Marketing, Carlson School of Management, University of Minnesota, 321 19th Ave. South, Minneapolis, MN 55455 (arao@csom.umn.edu). The first author is indebted to the ACR-Sheth Dissertation Grant Foundation for their financial support based on a dissertation proposal competitive award, to the Carlson School of Management for a competitive Doctoral Dissertation Fellowship, and to the University of Miami for its General Research Support Award and James W. McLamore Summer Awards. The authors also acknowledge the constructive comments of Terry Childers, Rajesh Chandy, Amna Kirmani, Kent Monroe, Michael Tsiros, Jerry Zhao, a seminar audience at the University of Colorado, Boulder, three anonymous reviewers, the associate editor, and the editor at *JCR* on earlier versions of this manuscript. Finally, we thank Lorena Bustamante for her help with data collection for study 3.

When evaluating the net impact of a series of percentage changes, we predict that consumers may employ a "whole number" computational strategy that yields a systematic error in their calculation. We report on three studies conducted to examine this issue. In the first study we identify the computational error and demonstrate its consequences. In a second study, we identify several theoretically driven boundary conditions for the observed phenomenon. Finally we demonstrate in a real-world retail setting that, consistent with our premise, sequential percentage discounts generate more purchasers, sales, revenue and profit, than the economically equivalent single percentage discount.

"The depression took a stiff wallop on the chin here today. Plumbers, plasterers, carpenters, painters and others affiliated with the Indianapolis Building Trades Unions were given a 5 percent increase in wages. That gave back to the men one-fourth of the 20 percent cut they took last winter."

The New York Times, quoted in *How to Lie with Statistics* (Huff 1954, 111).

INTRODUCTION

Percentages are frequently encountered in the marketplace. For instance, firms use percentages to communicate with a) consumers, when describing price changes or changes in product performance, b) investors, when describing financial information such as returns on investment, and c) public policy officials, when describing progress on meeting new regulations. Similarly, the government uses percentage information to communicate important changes in macroeconomic data, such as the rate of inflation or the growth in GDP, while followers of the stock market are often provided information about the daily change in popular indices such as the Dow Jones Industrial Average as a percentage gain or loss over the previous trading day's closing value. In a marketing context, multiple changes in numerical quantities such as price or product performance may also be expressed as multiple percentage changes.

Despite the ubiquity of such information in the marketplace, people often make mistakes in evaluating the consequences of a sequence of percentage changes. As demonstrated in the opening vignette, when assessing the impact of multiple percentage changes, the reporter mistakenly judged a 5% wage increase to be one-fourth, when it actually was one-fifth, of a preceding 20% decrease. Similarly, a 60% decrease followed by a 70% increase (resulting in a net decrease of 32%) on the standardized test scores in the state of California seemed to cheer up a lot of people (Dewdney 1993, 9-10). Such errors have obvious implications for marketing and consumer behavior. For example, if consumers mistakenly judge a 40% price discount followed by another 40% discount to be a total discount of 80% (Paulos 1988, 122), they might purchase more than they would have if the merchant had provided a single (economically equivalent) percentage discount of 64%. Retailers (e.g., *Macy's*, *J. C. Penney*, and *Saks Fifth Avenue*). frequently use the strategy of double discounts for their regular promotions or to induce customers to open a credit card account with them. Such errors in peoples' judgments of the net effect of multiple price discounts on the same product or on different products in a bundle, and of the sequential improvements in product features (e.g., the total improvement in fuel efficiency offered by the latest hybrid model over a traditional car) have implications for a variety of marketing settings including advertising, promotion, pricing and public policy. This computational error and its various consequences are the topic of the research reported in this paper.

The existing developmental literature in psychology has examined the difficulty that individuals have with mathematical computation in general (Ashcraft 1992, Gallistel and Gelman 1992; Parker and Leinhardt 1995; Pelham, Sumarta, and Myaskovsky 1994). In the marketing and consumer behavior literature, while researchers have recently started to examine the issue of consumer literacy and numerical competence in the marketplace (Adkins and Ozanne 2005; Viswanathan, Rosa, and Harris 2005), little research has examined the difficulties consumers have in processing percentage information (though see Heath, Chatterjee, and France 1995; Chatterjee et al. 2000 for exceptions). We extend these early tests of percentage processing by identifying a specific error people exhibit when they encounter a series of percentages, and demonstrating the implications of the error in both laboratory and real-world settings.

The rest of the article is organized as follows. We first examine the literature that describes the difficulties associated with the processing of percentage information. Based on this review, we develop a simple mathematical model and identify four mutually exclusive and collectively exhaustive manifestations of the computational error. The error and its consequences on attitudes and purchase intention are empirically demonstrated in our first study. In a second study, we circumscribe the phenomenon by examining several boundary conditions. Finally, to assess the impact of the error on actual purchase behavior, we test the prediction that double discounts should be perceived as a deeper discount than an economically equivalent single discount, in a field experiment. We observe that double discounts generate more purchasers, sales, revenue and profit than an economically equivalent single discount. We conclude with a discussion of the potential contributions of the research for theory and practice.

CONCEPTUAL DEVELOPMENT

The Problem with Percentages

Percentage calculations have been shown to be difficult for children (Hunting and Sharpley 1988), college freshman (Guiler 1946), and even mathematics teachers (Fisher 1988). Like similar difficulties with fractions and decimals, these difficulties have been explained by "whole number dominance", the notion that the mental representation of numbers may have developed in a way that favors whole numbers relative to decimals, fractions, percentages and other complex numerical forms (Behr, Post and Wachsmuth 1986). Consistent with this thinking, Saxe (1981) finds that people in the primitive Oksapian culture use different parts of their body (e.g., fingers) to represent numerosity, which leads to a mental numerical system dominated by whole numbers (Wynne 1997) Indeed, whole number dominance is a defining characteristic of the popular mental counting models in this literature (Mix, Levine and Huttenlocher 1999; Gallistel and Gelman 1992).

There are other, related explanations for whole number dominance. For example, Cosmides and Tooby (1996) argue that for evolutionary reasons, knowledge of whole numbers is probably more useful than knowledge of the more complex numerical forms for both predator and prey. Whole number dominance may also be due to the fact that "natural numbers precede rational numbers historically, mathematically (in most presentations), and psychologically" (Smith 1995, 5), though the direction of causality is difficult to determine. While the debate on what leads to whole number dominance is ongoing, whole number dominance seemingly leads to errors in computations involving fractions (e.g., 5/6 + 4/7 = 9/13, Behr et al. 1985; Bezuk and Cramer 1989), decimals (e.g., .17 > .7 because 17 > 7, Hoz and Gorodetsky 1989), and percentages (Venezky and Bregar's (1988) college student subjects failed to notice the asymmetry in percentage increases and percentage decreases; see also Guiler 1946; Parker 1994, 1997). In fact, percentages may be even harder to learn than decimals and fractions (Gay and Aichele 1997), because a percentage is unique in the sense that it can be used as either a number or as a function (Davis 1988), and percentage operations are fundamentally different depending on whether percentages are used as numbers or as functions. According to Bettman, Johnson and Payne (1990; Chase 1978), when percentage is used as a function denoting a relationship between two numbers, people "…must expend more cognitive effort … because this requires a multiplication operation or both multiplication and addition … (and) because multiplication operations typically require significantly more cognitive effort than addition operations"

(Morwitz et al. 1998, 456). Consistent with this argument, Chatterjee et al. (2000) find that mistakes with percentages are more prevalent among low relative to high need-for-cognition respondents. Because of the increased complexities associated with percentages, whole number dominance may be even more prevalent when people are asked to calculate with percentages.

When percentage is used as a mathematical function that denotes a specific relationship between two numbers, the specific quantity associated with a percentage depends on its base value. Two percentages that are associated with different base values have different weights and thus cannot be directly combined. Due to whole number dominance, however, people may mistakenly apply a simple whole-number strategy and add up the individual percentages directly. This misuse of a whole-number strategy will lead to a systematic computational error in how people process sequential percentages, as we discuss next.

Computational Error in Processing Sequential Percentages

In our context of a series of percentage changes, percentages are used as functions relating the magnitude of a change to the magnitude of a base. Therefore, two percentages in a sequence ought not be directly added to determine the net effect of the two changes. Simply put, a 20% discount on a $100 price followed by an additional 25% discount yields a final price of $60 (i.e., the first discount lowers the price to $80, and the additional discount yields a $20 decrease), implying an effective discount rate of 40%. Due to whole number dominance people may mistakenly add up the two discounts (i.e., 20% + 25%) and perceive the total discount to be 45%. More generally, since the real *net effect* of a sequence of changes differs systematically from a simple sum of the individual percentages (i.e., the *face value* of the sequence), such computational errors, when they occur, will produce predictable errors in peoples' judgment of the overall impact of the sequence.

To understand the consequences of this computational error, we draw from the analogous literature on how people evaluate multiple outcomes, a topic that has attracted the attention of behavioral scientists for the past two decades (Kahneman and Tversky 1979; Thaler 1980, 1985; Thaler and Johnson 1990; Prelec and Loewenstein 1998; Gourville and Soman 1998; Chen and Rao 2002). We follow Thaler (1985) and consider four possible mutually exclusive and collectively exhaustive outcomes when two percentage changes occur in a sequence: a) two increases in percentage, one after the other (*pure increases*), b) two decreases in percentage, one after the other (*pure decreases*), c) a percentage increase followed by a percentage decrease (or vice versa), where the combined effect of the two changes yields a real positive outcome (*a mixed increase*) and d) a percentage increase followed by a percentage decrease (or vice versa), where the combined effect of the two changes yields a real negative outcome (*a mixed decrease*). In the following section, a simple mathematical model is set up to better understand the nature of the error that may occur in each of the four scenarios.

A Simple Mathematical Model

Without loss of generality, let $v > 0$ be the original base value, and let a and b be the first and second percentage changes. For nontrivial cases, we have $a \neq 0\%$ and $b \neq 0\%$. The net effect of the two sequential changes is measured by the overall percentage change from the base value:

$$\textit{Net effect (NE)} = [v\,(1+a)(1+b)-v]/v = a + b + ab \tag{1}$$

If people mistakenly apply the whole number strategy, they will judge the overall effect of the sequence to be its face value (i.e., the sum of the individual values):

$$\textit{Face value (FV)} = a + b \tag{2}$$

It is apparent that $NE = FV$ only when $a = 0$ or $b = 0$. For a non-trivial sequence of percentage changes, the magnitude of the error created by the erroneous compounding is captured by the difference between the face value and the net effect, which is:

$$\gamma = FV - NE = a + b - (a + b + ab) = -ab \tag{3}$$

When the computational error occurs, it is straightforward to show that a series of pure increases (e.g., a 30% increase followed by a 25% increase) will be underestimated (i.e., as 55% vs. a real net increase of 62.5%), a series of pure decreases (e.g., a 30% decrease followed by a 25% decrease) will be overestimated (i.e., as 55% vs. a real net decrease of 47.5%), a mixed increase (e.g., a 40% increase followed by a 25% decrease) will be overestimated (i.e., as 15% vs. a real net increase of 5%), and a mixed decrease (e.g., a 25% increase followed by a 40% decrease) will be underestimated (i.e., as 15% vs. a real net decrease of 25%). Formal derivations are provided in Appendix A.

However, how consumer attitudes or behavior change due to the under- and over-estimation of the overall effect depends on the *valence* associated with the changes. For instance, the computational error will lead to an overestimation of double discounts, and since price decreases are favorable from the consumer's viewpoint, the overestimation will *enhance* purchase behavior more, relative to a single price discount of the same magnitude. On the other hand, depreciation of a new car's value presented as a sequence of percentage declines will also be overestimated, but since depreciation is unfavorable from the consumer's viewpoint, the overestimation will *dampen* purchase behavior more, relative to a single depreciation of the same magnitude. Following this logic, we predict that consumers' attitude towards the offer and purchase intention will differ depending on whether they encounter multiple or economically equivalent single percentage changes in the following manner:

H1: Pure increases and a mixed decrease that are associated with an *unfavorable* outcome (such as a net price increase), and pure decreases and a mixed increase that are associated with a *favorable* outcome (such as a net price decrease), will lead to a more positive attitude towards the offer and greater purchase intention relative to a single percentage change.

H2: Pure increases and a mixed decrease that are associated with a *favorable* outcome (such as a net increase in fuel efficiency), and pure decreases and a mixed increase that are associated with an *unfavorable* outcome (such as a net decrease in fuel efficiency) will lead to a less positive attitude towards the offer and lower purchase intention relative to a single percentage change.

Since the midpoint of the scales (i.e., 4) reflects indifference between a single percentage change and multiple percentage changes, the above predictions can be expressed in terms of how attitude towards the offer and purchase intention differ from 4 when people are asked to compare the multiple changes with the single change (see table 1).

Insert table 1 about here

We next turn to the empirical studies designed to test these predictions.

STUDY ONE

The existence of the computational error and its behavioral consequences as specified in hypotheses 1 and 2 were first assessed by asking participants to compare the effect of two sequential percentage changes (pure and mixed increases and decreases that were either favorable or unfavorable) with that of a single, arithmetically equivalent percentage change. To enhance generalizability, we replicated the study across two contexts describing changes in fuel efficiency and price respectively. A description of the stimuli in each cell, the specific percentages used in each cell, and the associated testable hypotheses, are presented in table 2 (see figure 1 for a sample stimulus corresponding to cell N in table 2). Different cover stories were used to accommodate the diversity of the stimuli in the two settings.

The use of different cover stories is to increase the realism of the stimuli. For example, we used decreases in gasoline price for beneficial decrease conditions, gas price increases for harmful increase conditions, depreciation in a car's value for harmful decrease conditions, and increases in a mutual fund's price as beneficial increases. The manipulation cannot be achieved realistically with the same cover story because, for instance, from the consumer's standpoint, an increase in the price of gas can not realistically be framed as "beneficial". In the analyses below, while the use of different cover stories is a potential confounding concern for the comparisons of cell means across different experimental conditions, it is not a concern for comparisons of each cell mean with the normatively correct answer (i.e., the mid-point of the scale).

Insert table 2 and figure 1 about here

Participants and Dependent Variables

Participants were recruited from introductory marketing classes at a major U.S. university, and were randomly assigned to each of the sixteen experimental conditions. Except for one cell (n =15), all other cells had 16 participants. The experiment was conducted on computers, and we used publicly available software (DeRosia 2000) to create the Web pages.

There were several dependent measures in all conditions. Each dependent variable appeared on a separate web page, and the web instrument was designed so that the participants could not go back and forth. We employed a five-item scale modified from Burton and Lichtenstein (1988) to measure participants' attitude towards the offer (AO) for one product relative to the other product, after stipulating that the products did not differ on any dimension other than the dimension that was manipulated (the scale was unidimensional: eigen value = 4.2, variance explained = 84%, and reliable: Cronbach's α = .95). Additionally, a separate single-item scale was employed to measure purchase intention (PI). The mid-point on all scales (i.e., 4 on our seven-point scales) was anchored as "the same" or "indifference", which is the arithmetically correct response. Following the PI question, an open-ended question elicited participants' reasons for why they answered the earlier questions as they did.

To assess the existence of the computational error, we also asked participants to indicate the net value of the sequence of percentage changes by responding to a multiple choice question containing four options: an option that was the arithmetic sum of the two percentage changes (representing the "computational error" option), the correct answer, an incorrect answer using another number that appeared in the stimulus (the "other error"), and a fill-in-the-blank option (the “other” choice). The order of appearance of the option reflecting the computational error and the correct answer was randomly varied across conditions. The multiple-choice format was chosen based on a pretest result which showed that using an open-ended format increased noise in people’s responses (i.e., quite a few participants provided random, but nevertheless erroneous answers). This multiple choice question appeared as the last dependent measure in all experimental conditions, and thus was always answered after the other dependent variables. (See appendix B for the dependent variables corresponding to the stimulus in figure 1).

Overall Results

Recall that we are interested in the degree to which responses deviated from the mid-point, *within each cell*. In general, we do not make predictions regarding the magnitude or direction of deviation due to particular factors such as the context or whether the percentage changes represented increases/decreases, and so on. In fact, we offer very specific predictions for each cell (see table 1). Nevertheless, we conducted omnibus tests, including MANOVA and ANOVA, as well as planned contrasts cross different experimental conditions, and found the results to be consistent with our predictions, though they are subject to confounding due to the use of different cover stories in different conditions. Therefore, we do not discuss these overall results further.

Insert table 3 about here

Key to our predictions was the planned contrasts we conducted to test if each cell mean was different from the midpoint of the scale. The results, reported in table 3, showed that all cell means are in the predicted direction, and in 14 out of 16 instances it was different from the midpoint of the scale ($p < .01$). The exceptions are AO in the pure favorable increase condition and PI in the mixed unfavorable decrease condition (not statistically different from the midpoint of the scale, $p > .20$). Overall, the results are largely supportive of our predictions.

Process Analysis

After showing that AO and PI do differ across different experimental conditions and differ from the normatively correct answer within each experimental condition in the predicted manner, we now turn to establishing a more direct link between the computational error and people's attitude and purchase intention. Towards that goal, we first examined participants' response to the multiple-choice question. A multinomial logit regression with five factors (with the question order as the fifth factor) revealed no significant differences in the accuracy / computational error ratio across experimental conditions or question order ($p > .10$). Overall, across the two contexts, a large proportion of participants (i.e., 59%) erroneously added percentages without recognizing that the first percentage change shifts the base. This compares to 26% of the participants who selected the correct answer.

In addition, planned contracts comparing each cell mean with the midpoint of the scale revealed that for the error-present groups, in all 16 condition, both AO and PI were in the predicted direction and significantly different from the midpoint of the scale ($p \leq .05$ or better). For example, consistent with hypothesis 2, in the pure favorable increase condition, the error-present group's attitude and purchase intention were smaller than the midpoint of the scale (3.51 < 4 for AO, $p = .05$; 2.24 < 4 for PI, $p < .01$). In contrast, for the error-absent group, in 14 out of the 16 conditions neither AO nor PI was different from the midpoint of the scale ($p > .10$ or worse). AO for pure harmful decreases and PI for mixed beneficial decreases were different from the midpoint of the scale ($p < .01$) even for the error-absent group. Therefore, in most cases there was a direct link between the presence/absence of the error and respondents' attitude and purchase intention.

Finally, to understand why respondents made the computational error, the responses to the open-ended question that attempted to elicit subjects' reasoning for their responses to the attitude and purchase intention measures were divided into three mutually exclusive categories. The first category contained responses from those who displayed a correct understanding of the arithmetic of multiple percentage changes, including all participants who performed the correct calculation, *or* mentioned the interdependent nature of the two sequential changes in the stimuli, *or* simply mentioned that the sequential change was the same *or* about the same as the single change. Forty-six (i.e., 18%) responses fell into this category reflecting "correct" reasoning. A second category comprised individuals who justified their responses by demonstrating the misuse of the whole-number strategy of adding up the multiple percentages (e.g., "If it depreciates by 40% in the first 5 years (8% per year), that is LESS than 10% per year for 5 years (10%*5=50%)" for cell L in Table 2). One hundred thirty one (i.e., 51%) responses fell into this category reflecting the computational error. The last group of 78 (i.e., 31%) consisted of missing data and responses that appealed to factors other than arithmetic to explain their response. As shown in Table 4, participants' responses to the multiple choice question and their responses to the open-ended "why" question are statistically associated ($\chi^2 = 121.17$, $d.f. = 1$, $p < .0001$; $\chi^2 = 151.04$, $d.f. = 1$, $p < .0001$ when the "Other" category was removed from both questions). So consistent with our theory, it seems that the computational error is indeed driven by the mistaken use of the whole number strategy of adding up multiple percentages.

Insert table 4 about here

Discussion

This study provides direct evidence documenting the existence of the computational error among a large proportion of study participants. Further, there is a systematic and predictable under- or over-estimation of the net impact of a sequence of percentage changes, such that attitude toward the offer and purchase intention for the product or service undergoing the sequential changes differed systematically with how the percentages are framed (i.e., the direction, type, and valence of changes), and differed from those undergoing an economically equivalent single change, in a manner that is consistent with the existence of the computational error. The results are robust across two different contexts. In addition, we were able to link the variations in attitude and purchase intention with the absence/presence of the computational error, and link the error to the inappropriate employment of the whole-number strategy in adding up multiple percentages.

While the results of study one provide support for the existence of the computational error and its marketing consequences, a plausible rival explanation for our result relies on a mental accounting mechanism. For example, when people are presented with sequential percentage increases in a favorable attribute and the economically equivalent single increase, people may prefer the former to the latter, perhaps because of the "segregation of gains" principle (Thaler 1985; though they do not spontaneously and optimally integrate or segregate when given the opportunity to do so, Thaler and Johnson 1990; Linville and Fischer 1991; Thaler 1999). However, if mental accounting principles operated, half of our predictions would *not* have been supported. For instance, a mental accounting perspective would predict that multiple losses that are integrated should be preferred. In contrast, our test of H1 indicates that multiple losses (unfavorable increases), when *segregated*, yield enhanced attitude and purchase intention for people who make the computational error. Similarly, our results concerning pure beneficial decreases (hypothesis 1), mixed beneficial increases (hypothesis 1), and mixed harmful increases (hypothesis 2), are opposite to the mental accounting principles on segregating multiple gains, combining mixed gains, and segregating mixed losses (silver lining), respectively. In addition, when the error was absent, respondents in study one were mostly indifferent between two economically equivalent outcomes that were framed differently, suggesting that in our context of sequential percentage changes on the *same* product, mental accounting principles may not have been operative. In a follow-up study not reported here, we directly manipulated the ease of integration or segregation of multiple percentage changes, and observed that the computational error we identified here did operate independently of the mental accounting principles (Details of this study can be obtained from the authors).

The results of study one show that many participants made the computational error of adding up multiple percentages, yet other participants were accurate in their judgment. The co-existence of the error-present and error-absent groups suggests that some individual or situational factors may drive the manifestation of the computational error. In the next set of studies, we examine this issue and identify some boundary conditions of the error. We demonstrate that the error rate varies with people's motivation, the difficulty of the calculations, and the face validity of the answer associated with the computational error.

STUDY TWO

The set of studies we report under the rubric of study two are designed to identify boundary conditions for the computational error identified in study one. Particularly, since the computational error does not appear to be a universal phenomenon, we were interested in identifying the conditions that attenuate the error. For instance, one possible explanation for the manifestation of the error is that though people know the appropriate arithmetic rules, they make an effort-accuracy tradeoff in choosing their calculation strategies (Payne, Bettman and Johnson 1993). In other words, people may not perform the correct calculations because the effort required is deemed to be too high, or the benefit of calculating the correct answer is deemed to be too low. Based on this argument, we can potentially reduce the error rate by increasing people's motivation to carry out the correct calculations, or by reducing the computational complexity of the task. Another way to assess whether an effort-accuracy tradeoff is responsible for the observed error is to alert participants to the fallacy of directly adding up percentages. For example, when the answer is fallacious, people may realize that arithmetically combining percentages is inappropriate, and they may therefore become more careful and more accurate.

In the following three studies, we test the effects of motivation, ease of calculation and the fallacious outcomes, on people's error rate and accuracy. The first two studies are about shopping for a textbook on the Internet, and they are identical except for the specific manipulations. The cover story for those two studies describes two sequential percentage discounts offered by an online store, and respondents are asked to judge the total percentage discount offered by this store. To avoid confusion, participations were explicitly told, for example, that "the sale price is 30% below the list price. In addition, there is a special promotion going on that allows you to save an additional 25% off of the already reduced sale price". Similar to study one above, to reduce randomness in responses, a multiple-choice format was employed to elicit participants' assessment of the correct answer. That question offered three choices: the correct answer, the answer that reflects the computational error, and an "other (please specify)" choice. The "other error" option from study one was dropped because only 3.5% of respondents picked that option in study one. Study 2c is similar to studies 2a and 2b, but to make the large percentage increases and decreases credible, it describes fluctuations in gasoline prices. The order of the correct answer and the one reflecting the computational error, which is counterbalanced in all studies, does not significantly affect the results ($p > .10$), and is therefore not discussed further.

Study 2a: The Role of Motivation

In this study, we examine how people's motivation to be accurate affects the manifestation of the computational error. We expect that the error rate will decrease and accuracy will increase when people are motivated to figure out the total percentage discount. To test this possibility, we manipulated respondents' motivation by offering a monetary incentive of $2 for the correct answer in one condition, and no incentive in the other condition. The percentages used in both conditions are identical to those in cell J of table 1. One hundred twenty seven undergraduate business students enrolled in introductory marketing classes at a major U.S.

university participated in this study for extra course credit. Participants were randomly assigned to one of the two conditions. The respondents answered the multiple-choice question (i.e., Question 1), and two additional questions measuring their motivation (i.e., “I was highly motivated to answer Question 1 accurately” and “There was not enough incentive for me to work hard on Question 1”). Finally they provided demographic information.

Results. After reverse coding the second item, the two motivation questions were significantly correlated ($r = .306$, $p < .0001$) and the average measure showed a successful motivation manipulation (4.70 > 4.21, $t_{124} = 2.04$, $p < .04$). A multinomial logit shows that the $2 incentive increased the accuracy/error ratio ($p < .05$). The rate of the computational error dropped from 44% to 26% based on a *z*-test ($z = 2.08$, $p < .04$), and accuracy increased from 41% to 56% ($z = 1.63$, $p = .10$, directionally consistent with the prediction). When people are motivated to carry out the calculations, they are less likely to make the error and more likely to be accurate in calculating the total effect of sequential percentage changes.

Study 2b: The Role of Calculation Difficulty

As discussed above, instead of increasing peoples’ motivation, we may reduce the error rate and improve accuracy by making calculations easier. Therefore, in this study, we manipulate the difficulty of performing calculations by providing two easy percentage discounts in one condition and two difficult but otherwise similar percentage discounts in the other condition. In the easy condition, the percentage discounts are 50% and 20%, and in the difficult condition, 55% and 15%. We include two original base prices, $100 and $80, to test the robustness of the results. Therefore, we have a 2 (calculation difficulty: high vs. low) by 2 (base price: $100 vs. $80) between-subjects design. One hundred twenty six students from the same subject pool as in study 2a participated in the study for extra course credit. Participants were randomly assigned to one of four conditions. Cell size varied from 29 to 34. Participants answered the multiple-choice judgment question (i.e., Question 1), followed by two questions measuring the easiness of the task (i.e., "Figuring out the answer to Question 1 was an easy task" and "The percentages encountered in the store are easy percentages"). Finally, they provided demographic information.

Results. The two questions used to measure the easiness of the calculations are positively correlated (r = .412, p = .000) and are averaged as an easiness measure. A two-factor (calculation difficulty and base price) ANOVA revealed a significant effect of the calculation difficulty factor (p = .000; p > .61 for all other effects) on the easiness measure. A planned contrast showed that the manipulation worked as intended (5.59 for easy condition > 4.59 for difficult condition, on a seven-point scale, p < .05). A multinomial logit on accuracy with the two factors, revealed that the only significant effect was that of calculation difficulty. The accuracy/whole-error ratio was higher (p = .003; p > .17 for all other effects) when the calculations were easy. The error rate dropped from 38% to 19% (z = 2.31, p = .02), and accuracy increased from 43% to 79% (z = 4.14, p = .000), from the difficult to the easy conditions. Therefore, it appears that people are less likely to display the computational error and more likely to be accurate when the calculations are easy.

Study 2c: Face validity

In this study, we examine how the face validity of the answer that is associated with the computational error affects the error rate and accuracy. Specifically, we predict that when the computational error leads to an answer that is illogical, people will easily recognize the fallacy of directly adding up the two percentages, and this recognition may improve their accuracy and they may avoid the obviously erroneous answer. To manipulate the amount of effort required to recognize the fallacy of the computational error, we presented respondents in one condition with two large percentage *increases* in prices (70% and 45%), while respondents in the other condition were exposed to percentage *decreases* of the same magnitudes. We predict that the error rate will decrease and accuracy will increase in the decrease condition, where the computational error will lead to an illogical answer, for example, a decrease of 115% in the price. Forty-six students from the same subject pool as in study 2a participated in this study, with 24 in the increase condition and 22 in the decrease condition. To make it credible, the cover story for this study described changes in gasoline prices. Not surprisingly, respondents perceived the increases to be more believable than the decreases in gasoline price (3.33 vs. 2.32 on a seven-point scale, $p < .05$), but believability does not mediate (cannot explain) the predicted effect ($p > .50$ for the covariate and $p < .05$ for the predicted effect, when believability is used as a covariate in the multinomial logit reported below).

Results. A multinomial logit revealed a significant effect of increase/decrease on the focal judgment question ($p < .05$). Compared with the increase condition, the decrease condition yielded fewer errors (18% < 50%, $z = 2.26$, $p < .05$) and a higher level of accuracy (55% > 29%, $z = 1.75$, $p < .10$, directionally consistent with the prediction). Presumably, the illogical answer associated with the computational error alerted people to the fallacy of directly adding up the two percentages, as a result of which they made fewer errors and improved their accuracy.

In this series of three studies, we identified some theoretically driven boundary conditions for the computational error. We find that the error decreases (and accuracy increases) when people are motivated to carry out the correct calculations, when the calculations are easy, and when the fallacy associated with directly adding up percentages is obvious. Seemingly, an effort-accuracy trade-off may be occurring for some people. Note here we are equating a reduction in the computational error with an increase in accuracy. However, this may not always be the case. In a follow-up study, for example, we found that with an increase in people's numerical ability, the computational error decreases, but people's accuracy first increases then decreases, suggesting that while novices make the computational error, experts may use the wrong answer as an approximation for the correct answer (Details of this study can be obtained from the authors).

Since this computational error can potentially influence peoples' judgment in a variety of settings, the economic impact of such errors on consumer welfare may be substantial. Therefore, an assessment of whether the computational error leads to differences in *actual* behavior is likely important. We address this issue in study three.

STUDY THREE

We chose double discounts as a context in which to examine the real-world consequences of the computational error. When faced with double discounts, consumers who erroneously employ a whole number computational strategy will likely overestimate the impact of the discount. Therefore, consistent with H1, double discounts will be perceived to be deeper than a single discount of the same economic value, and consequently ought to induce more purchases and yield commensurate economic benefits to the firm.

To examine this effect in a natural setting, we ran a controlled experiment in a retail store, varying the form of discount (double or single). We reasoned that the number of purchasers, sales, revenue and profit would be higher during the periods in which double discounts were offered, relative to when the economically equivalent single discount was offered. We were afforded the opportunity to manipulate price promotions on a selected set of products in a small local retail store. We were also given access to their revenue and profit data for the promoted products as well as for the entire store, which enabled us to directly examine the economic impact of the computational error and rule out competing explanations for the observed effect.

Store and Product Selection

The site for our study was a small upscale kitchen appliance store that is located on the main street of a small and wealthy town in the southeast part of the U. S. (population: around 50,000; median household income: more than $80,000; education level: over 95% with high school, over 50% with a bachelor's degree or better, and over 20% with a Master's degree or better, according to the 2000 census). Twelve *Totally Bamboo* cutting boards were selected as the focal products. These products are moderately high priced (average price = $46; median price = $38). Our reasoning for selecting this product line was that while a discount on an inexpensive product may not be particularly effective at increasing sales, very expensive products may move too slowly for us to observe any effect in the short run. There had been no other promotional activity in the focal category all year. In addition, during the promotion periods, all other activity in the store (number of salespeople, other promotions and the like) remained stable.

Design of the Study

Based on consultation with the store owner, we offered 40% as the single discount, and a 20% discount and an extra 25% discount as the corresponding double discounts. The two discounts are economically identical. We chose these specific percentages because they are frequently encountered in this market and thus should have face validity. In addition, the choice of the percentages was made to (a) offer customers a reasonably deep discount in order to maximize the chance of observing the effects of the price promotions; and (b) avoid any ceiling effect associated with extremely deep discounts. The type of discount was manipulated over time. As dependent variables, we recorded the number of purchasers, sales volume, revenue and profit for the thirteen products, on a daily basis. We also recorded the total number of purchasers, sales volume, revenue and profit for all other products in the store, as proxies for store traffic.

Data Collection

We first ran price promotions on the selected products from April 4, 2005 to April 30, 2005, offering the single discount for the first two weeks and the double discounts for the next two weeks. To counterbalance the order of the two types of discounts, we then ran the same promotions again from September 12, 2005 to October 8, 2005, this time offering the double discount for the first two weeks and the single discount for the next two weeks. The selection of the time windows was based on the fact that there was no major holiday during or close to the study periods. The store was open Monday through Saturday in each of the eight weeks. Thus, we obtained 24 days of data for the single discount and double discounts, respectively.

Analysis and Results

The two data series, when lined up by week and day of the week (e.g., Monday of the first week for the single discount with Monday of the first week for the sequential discount, etc.), were highly correlated ($r = .60$, $p < .005$ for number of purchasers, $r = .69$, $p < .001$ for sales volume; $r = .69$, $p < .001$ for revenue; $r = .68$, $p < .001$ for profit). These high correlation coefficients suggest systematic variations due to day of the week. To control for these variations, we treated data from each day of the week as our unit of analysis, and treated the order of the discounts, the types of discounts and the number of week as independent variables in a repeated measures ANOVA, which revealed significant effects of the type of discount on number of purchasers ($p < .08$), sales volume ($p < .02$), revenue ($p < .10$) and profit ($p < .04$) respectively; no other effects were significant ($p > .20$). The number of purchasers, sales volume, revenue and profit on the focal products were all higher when the double discounts were offered than when the economically equivalent single discount was offered ($p < .05$ or better; Actual values of number of purchasers, sales volume, revenue and profit for the two periods can be obtained from the authors).

While the results are consistent with our predictions, since the different types of discounts were offered in different weeks, we need to ensure that the observed effects are not due to some uncontrolled store-specific or environmental factor that varied over time such as changes in weather conditions. When we compared the four weeks when the single discount was offered with the four weeks when the double discounts were offered, the total number of purchasers, sales volume, revenue and profit on the remaining products in the store stayed stable ($p > .24$ or worse), suggesting that the overall store traffic likely did not vary. To more rigorously control for variations in overall store traffic in our analysis of the focal products, we computed the number of purchasers, sales volume, revenue and profit of the promoted products as *proportions* of the total number of purchasers, sales volume, revenue and profit in the store, and found support for our predictions when the four proportions were used as dependent variables (single vs. double discount periods ($p < .05$ or better).

Furthermore, our data also showed an increase in the number of promoted products purchased per customer (i.e., the sales volume for the promoted products divided by the total number of purchasers in the store, $p < .001$). While this result is consistent with the existence of the computational error (i.e., a customer would buy more when s/he perceives that a larger discount is offered), it cannot be readily explained by an increase in overall store traffic. Finally, the results may also be explained by the mental accounting mechanism of segregating multiple gains. While we do not have data from this study to directly speak to this issue, as we discussed earlier in relation to the results from study one, mental accounting principles cannot be operative in our context of sequential percentage changes on the same product. That means our results in the field study are unlikely to be driven by a mental-accounting mechanism. Therefore, while we realize that any conclusion based on such a small-scale study is tentative, the data replicates the results of our lab experiments, and are indeed in line with our computational error based explanation.

GENERAL DISCUSSION

In three studies employing a variety of stimuli and methodologies, we demonstrate the existence of a systematic and predictable computational error when people encounter a series of percentage changes. We argue that this error is a consequence of the inappropriate application of whole-number computational rules to percentages, and that it has predictable attitudinal, behavioral-intention, and purchase behavior consequences.

We contribute to the literature on the processing of percentages in various ways. *First,* we provide a formal model to examine the manner in which the provision of percentage information in the marketplace is subject to erroneous interpretation. In a host of settings ranging from changes in prices to the performance of a financial portfolio, the presentation of the information in percentage format provides substantial opportunity for the computational error to reveal itself. The model allows us to identify a particular computational error that some people may exhibit when they process multiple percentage changes. *Second,* we identify three important moderators that may reduce the manifestation of the error: when people are motivated to compute the correct value, when calculation is easy, and when the erroneous heuristic yields an obviously fallacious result. *Finally,* we show the consequences of this error on sales and profits of merchants who may employ a strategy that capitalizes on the error. That is, the error allows information purveyors to be strategic in how they present numerical information, and therefore has important marketing and public policy implications regarding the manner in which sequential percentage changes ought to be communicated. We expand upon the implications of our research below.

Practical Implications

The provision of price change information in numerical form can be accomplished in absolute terms or as a percentage change (Chen, Monroe, and Lou 1998), and are subject to the computational error described when presented in percentage terms. In addition, other numerical information such as changes in product performance, nutrition information, the degree to which a new technology performs relative to older technology, the performance of financial markets, changes in macroeconomic indicators, and reductions and/or increases in corporate as well as government budgets, are just a few settings in which information is frequently presented as a series of percentage changes. And, the audiences for these messages range from lay consumers and investors to sophisticated mutual fund managers and the United States Congress. To the extent that these audiences make the computational error, they may incur substantial economic costs. The consumer welfare consequences of the error have obvious public policy implications.

To the extent that dishonest purveyors of information employ the presentation of sequential percentage changes as a means of deceiving their audience, the issue ought to be of interest to regulatory agencies. For instance, some financial service firms may exploit the error by presenting performance information in a manner designed to make the client's portfolio appear better than it really is. Similarly, if the computational error contributes to consumers' abuse of revolving credit (De Graff, Wann, and Naylor 2001, 18-22, 212-213), regulatory agencies may have another argument to require credit card companies to explicitly state the net impact of compound interest rates over the long haul as a way of protecting consumer welfare.

Theoretical Implications

One consequence of the miscomprehension of percentage information is that economically equivalent options may be perceived differently depending on how they are presented, or "framed". However, the computational error identified in this research is fundamentally different from perceptual biases due to framing effects in the behavioral decision theory (BDT) literature. According to the BDT perspective people *evaluate* information differently from what traditional economic models postulate (Kahneman and Tversky 1979). In contrast, we suggest that people make a computational error in that they misapply whole-number computational strategies to percentages when they *encode* percentage information. Since these processes occur at different stages of information processing, the error may affect people's preferences independently of mental accounting (e.g., loss aversion). In this respect, our research is different from Heath et al. (1995), who were interested in identifying boundary conditions for Thaler's (1985) mental accounting principles.

Note, however, that our results are consistent with Heath et al.'s (1995) empirical reversal of the mental accounting principle for mixed gains that are presented in the percentage format. While mental accounting principles would predict that an outright gain (+$49) should be preferred to a mixed gain (+$99, and -$50), Heath et al. observe the opposite with corresponding percentages, i.e., a mixed gain (+33% which corresponded to a $99 price reduction on a $300 item, and -5% which corresponded to a $50 increase on a $1000 item) was preferred to an outright gain (+3.8% which corresponded to a $49 price reduction on a $1,300 item). One of the explanations proposed by Heath et al. for this reversal is a value function in which the abscissa consists of percentages. Similarly, in Chatterjee et al. (2000), the authors argue that people (especially those with low need-for-cognition) are likely to take percentages at their face value. Therefore, our thesis that people may mistakenly add up percentages as if they were whole numbers, is consistent with their general premise that people may take percentages at their face value.

Conclusions and Future Research

In this research, we identify a systematic and relatively widespread error in how people compute multiple percentage changes that has important marketing consequences. If the error is indeed driven by whole number dominance, we should expect to observe similar errors when consumers are presented with information in other complex numerical forms (e.g., fractions: "Buy one, get the second one at 1/2 off the original price"). More generally, to the extent that the computational error is related to the broader issue of innumeracy (Paulos 1988), we suspect that any information that requires calculation (e.g., nutrition information) may be subject to various errors. Given the increasing importance of numerical information in this information age, understanding the manifestation of similar errors and identifying mechanisms to correct them are of considerable theoretical and practical significance.

Another interesting avenue for future research is the relationship between the computational error and the notion of math anxiety. For instance, Tobias' (1995) argument that math anxiety may be related to the use of language in mathematics (e.g., "multiple" means increase in everyday language, but multiplication by a fraction may decrease a value) can be applied to the context of double discounts. For example, the use of "extra", "additional", or even the "+" sign in the wording of double discounts may induce people to add up sequential percentages. Thus, factors that contribute to math anxiety (e.g., language, spatial visualization abilities) may also affect the computational error, or whole number dominance in general.

Finally, when retailers announce the total discount to consumers (e.g., "Total savings of 45% off original prices when you take an extra 30% off") are double discounts more effective in conveying certain intentions of the retailer (e.g., the urge to clear out an item)? If so, what are the implications of such a message on price and quality perception? What roles do product features (e.g., search vs. experience products) and consumer characteristics (e.g., numerical experts vs. novices) play in these situations? These and related questions should be explored further in lab and field studies.

APPENDIX A

MODEL DERIVATION

From (3) in the text, we get:

$$FV = \gamma + NE \qquad \textbf{(4)}$$

Implication 1***:*** For a pure increase, $a > 0$ and $b > 0$, $NE > 0$ from eq. (1), $FV > 0$ from eq. (2), and $\gamma < 0$ from eq. (3). Therefore, $NE > FV > 0$ from eq. (4).

Implication 2***:*** For a pure decrease, $a < 0$ and $b < 0$, $v(1+a)(1+b) < v$. $NE < 0$ from eq. (1), $FV < 0$ from eq. (2), and $\gamma < 0$ from eq. (3). Thus, $FV < NE < 0$ from eq. (4).

Implication 3***:*** For a mixed increase, $a > 0$, $b < 0$ (or conversely $a < 0$, $b > 0$), and $NE > 0$, $\gamma > 0$ from eq. (3). Therefore, $FV > NE > 0$ from eq. (4).

Implication 4***:*** For a mixed decrease, $a > 0$, $b < 0$ (or conversely $a < 0$, $b > 0$), and $NE < 0$, $\gamma > 0$ from eq. (3). Therefore, either $0 > FV > NE$, or $FV > 0 > NE$, from eq. (4). If $FV > 0 > NE$, then, $a + b + ab < 0$ from eq. (1), and $a + b > 0$ from eq (2). Without loss of generality, let $a > 0$ and $b < 0$. Solving for these two equations, we get:

$$-a < b < \frac{1}{1+a} - 1 < 0 \qquad \textbf{(5)}$$

This suggests that, when eq. (5) is satisfied, $FV > 0 > NE$ (e.g., $a = 40\%$ and $b = -30\%$, $NE = -2\%$, $FV = 10\%$). This is an interesting scenario in which a net reduction may erroneously be encoded as a net increase, due to the computational error. When (5) is not satisfied, then $0 > FV > NE$ (e.g., $a = 20\%$ and $b = -30\%$, $NE = -16\%$, $FV = -10\%$).

APPENDIX B

DEPENDENT MEASURES FOR STIMULUS PRESENTED IN THE FIGURE

I. Attitude Towards the Offer Items

Recall that the two stations are equally close to your apartment. Thus, your attitude toward going to the two stations should be the same, *if* their gas prices are the same.

"Compared with filling up the gas at Station *A*, filling up the gas at Station *B* is (seven-point scale):

Favorable-Unfavorable
Bad-Good
Detrimental-Beneficial
Attractive-Unattractive"

Please indicate whether you agree or disagree with the following statement (seven-point scale):

"Compared with filling up the gas at Station *A*, I like the idea of filling up the gas at Station better

Strongly disagree – Strongly Agree"

II. Purchase Intention Measure
Recall that the two stations are equally close to your apartment. Thus, you should be indifferent between going to the two gas stations, *if* their gas prices are the same.

"How likely is it that you will drive to Station *B*, instead of Station *A*, to fill up your gas?

Very unlikely – Very Likely

III. Open-ended question

Please provide a detailed explanation as to why you answered as you did in the previous question.

IV. Accuracy Measure Options

What is the *overall* price decrease at gas station ***B*** from last week? (check one)

__ 40%
__ 25%
__ 15%
__ Other (please specify) ______ %

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TABLE 1

PREDICTIONS ON ATTITUTE TOWARDS THE OFFER (AO) AND PURCHASE INTENTION (PI) IN EACH EXPERIMENTAL CONDITION

		Increase	Decrease
Pure	Favorable	< 4	> 4
	Unfavorable	> 4	< 4
Mixed	Favorable	> 4	< 4
	Unfavorable	< 4	> 4

*: 4 reflects indifference between a single change and the multiple changes, a number larger than 4 indicates that multiple changes are preferred, and a number smaller than 4 indicates that the single change is preferred.

TABLE 2

STIMULI USED AND THEIR PREDICTED CONSEQUENCES (STUDY I)

Fuel Efficiency		Increase	Decrease
Pure	Favorable	**A.** Sequential improvement in MPG delivered {(30%, 25%) vs. 62.5%} will lead to an underestimation of benefit	**B.** Sequential reductions in fuel consumption {(-30%, -25%) vs. –48%} will lead to an overestimation of benefit
	Unfavorable	**C.** Sequential increases in fuel consumption {(30%, 25%) vs. 62.5%} will lead to an underestimation of harm	**D.** Sequential reductions in MPG delivered {(-30%, -25%) vs. –48%} will lead to an overestimation of harm
Mixed	Favorable	**E.** Sequential changes in MPG delivered {(-25%, 40%) vs. 5%} will lead to an overestimation of benefit	**F.** Sequential changes in fuel consumption {(25%, -40%) vs. –25%} will lead to an underestimation of benefit
	Unfavorable	**G.** Sequential changes in fuel consumption {(40%, -25%) vs. 5%} will lead to an overestimation of harm	**H.** Sequential reductions in MPG delivered {(-40%, 25%) vs. –25%} will lead to an underestimation of harm

Price Setting		Increase	Decrease
Pure	Favorable	**I.** Sequential improvement in mutual fund returns {(30%, 25%) vs. 62.5%} will lead to an underestimation of benefit	**J.** Sequential reductions in gasoline prices {(-30%, -25%) vs. –48%} will lead to a an overestimation of benefit
	Unfavorable	**K.** Sequential increases in gasoline prices {(25%, 30%) vs. 62.5%}will lead to an underestimation of harm	**L.** Depreciation of a car’s value {(five 10% declines) vs. -40%} will lead to an overestimation of harm
Mixed	Favorable	**M.** Sequential changes in Mutual Fund returns{(40%, -25%) vs. 5%} will lead to an overestimation of benefit	**N.** Sequential changes in gasoline prices {(25%, -40%) vs. -25%} will lead to an underestimation of benefit
	Unfavorable	**O.** Sequential changes in gasoline prices {(40%, -25%) vs. 5%} will lead to an overestimation of harm	**P.** Depreciation of a car’s value {(-40%, 25%) vs. –25%}will lead to an underestimation of harm

TABLE 3

CELL MEANS (S.D., SAMPLE SIZE) ATTITUTE TOWARDS THE OFFER (AO) AND PURCHASE INTENTION (PI) (STUDY ONE)

		Increase		Decrease	
		AO	PI	AO	PI
Pure	Favorable	3.94 (1.54, 32)	2.97 (1.66, 32)*	4.81 (1.64, 32)*	4.75 (1.93, 32)*
	Unfavorable	5.10 (1.62, 32)*	5.19 (1.97, 32)*	2.93 (1.61, 32)*	2.52 (1.84, 31)*
Mixed	Favorable	4.59 (1.35, 32)*	5.00 (1.53, 31)*	3.05 (1.81, 31)*	2.45 (1.73, 31)*
	Unfavorable	2.85 (1.53, 32)*	2.88 (1.95, 32)*	4.71 (1.50, 32)*	4.38 (1.64, 32)

*: different from 4, the mid-point of the scale ($p < .01$, based on t-tests that used the overall MSE from the repeated measure analysis and the associated degrees of freedom)

TABLE 4

CROSS-TABULATION OF THE MULTIPLE CHOICE QUESTION

AND THE OPEN-ENDED “WHY” QUESTION

(STUDY ONE)

		Multiple choice judgment question			
		Computational error	Correct	Other	Subtotal
Open-ended “why” question	Use of whole-number strategy	**113 (86, 75)**	**0 (0, 0)**	18 (14, 47)	131 (100, 51)
	Use of correct strategy	**1 (2, 0)**	**42 (91, 64)**	3 (7, 8)	46 (100, 18)
	Other	37 (47, 25)	24 (31, 36)	17 (22, 45)	78 (100, 31)
	Subtotal	151 (59, 100)	66 (26, 100)	38 (15, 100)	255 (100, 100)

* Frequency (row percentage, column percentage)

FIGURE 1
WEB STIMULUS FOR CELL N IN TABLE 1