Prominent Retailer and Intra-brand Competition

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Abstract

Online retail search traffic is often concentrated at a “prominent” retailer for a product. The authors unpack the ramification of this pattern on pricing, profit, and consumer welfare in an intra-brand setting. Prominence denotes a larger number of heterogenous search cost consumers starting their search at the prominent retailer than at any other retailer. This analyses show that search traffic concentration can intensify intra-brand competition, can lower average prices of all retailers, and can improve consumer welfare. Interestingly, the prominent retailer’s incremental traffic advantage can increase or reduce its own profit; the authors denote these as the “blessing” and “curse” of prominence respectively. The authors extend their analysis to a setting where consumers consider searching only amongst those retailers of whom they are individually aware of; the prominent retailer is included in all these individual awareness sets. The effects on market average prices and welfare carry over, but only below a critical threshold level of the prominent retailer’s first-search traffic advantage. Above this threshold, market average prices rise and welfare decreases, making this the region where search concentration warrant scrutiny from policy makers. The authors close with policy remedies, and managerial implications of search concentration.

Keywords: Search Traffic Concentration, Intra-brand Competition, Ordered Search, Search Cost Heterogeneity, Digital Markets, Competitive Strategy, Curse of Prominence

Online search traffic patterns show consumers conducting their searches in a selective manner concentrated on certain competitors. To illustrate, Amazon is the starting point for 44% of U.S. consumers when they go to purchase an item online (NPR 2018). In its legacy markets such as online books, 65% of consumers start their search at Amazon (De Los Santos, Hortaçsu, and Wildenbeest 2012). In this research, we denote the search traffic advantaged online retailer as the "prominent" retailer.[1]

Given the observed interest in driving traffic with search engine optimization (SEO) and other tactics, this search traffic advantage appears own-profit improving, perhaps even at the expense of competition and consumer welfare. For instance, former Treasury Secretary Mnuchin famously noted that "…. there's no question they (Amazon) have limited competition …" (Fitzgerald 2019). Antitrust scholars (e.g., Khan 2018) posit that search traffic concentration may be emerging as the chief antitrust concern of the digital age. Indeed, multiple agencies are presently conducting antitrust reviews of search traffic dominant firms (Wall Street Journal 2019; Subcommittee on Antitrust 2020). Despite these expressed concerns, there is limited systematic insight into the impact of search traffic concentration on competition or consumer welfare, making it difficult to frame an appropriate policy response.

We seek to contribute to closing this gap in a specific setting; viz. by unpacking the ramifications of search traffic concentration in an intra-brand setting,[2] where consumers search for better prices of the identical product across competing retailers. Specifically, we examine the following interrelated questions. At the product-market level, how does traffic concentration influence competition and consumer welfare? [3]At the firm level, how might a prominent retailer leverage its privileged traffic position in price setting? Might more traffic always improve a prominent retailer's profit?

In our game-theoretical model, competing retailers offer the same product to heterogenous search cost consumers, who acquire information about possibly better prices through sequential search. In our initial model, consumers are aware of all retailers, thus only one's personal (positive) search cost constrains that consumer from exhaustively searching each retailer in turn. More consumers start their search at the prominent retailer than at any other single retailer.

Counter-intuitively, we find traffic concentration *lowers* average market price and thus improves consumer welfare. The prominent retailer can potentially capture more high-search-cost consumers given its first-search advantage, which leaves fewer such consumers remaining for other retailers, forcing the latter firms to compete for a customer mix disproportionately made up of low-search-cost customers. This heightened competition puts price pressure on these latter retailers to lower their prices, making continued search more attractive to those consumers who start at the prominent retailer. In response to this pressure, the prominent retailer has to lower its own price to keep its first-search customers from searching further. In sum, this lowers prices at all retailers, due to search traffic concentration, improving consumer welfare.

Greater prominence is not always a blessing despite endowing the retailer to charge a higher price than its competitors. The "curse" of prominence is that its own profit decreases with increased first-search advantage. Intuitively, as above, the prominent retailer can potentially capture more high-search-cost consumers as its first-search advantage improves, but this simultaneously intensifies price competition, which weakens its ability to profit from these sales. The negative pricing effect dominates the positive demand effect when the prominent retailer already faces a high demand. In such a case, more first-search traffic lowers the prominent retailer's profit. In sum, counter-intuitively, search traffic advantages are not monotonically own-profit-improving.

We extend our analysis to consider a setting where online consumers limit their searches to relatively small subsets of retailers of which they are personally aware. These limited consideration subsets are a ubiquitous feature of consumer behavior models in marketing (e.g., see Hauser and Wernerfelt 1990; Nedungadi 1990; Amaldoss and He 2013; 2019). To illustrate, 90% of online book customers searched three or fewer online booksellers (De Los Santos, Hortaçsu, and Wildenbeest 2012) amongst the hundreds of sellers actively present in the market. However, the prominent retailer is almost always included in these individual awareness sets to search; e.g., 92% of US online book consumers visit Amazon (NPR 2018) at some point on their search journey.

We assume that the prominent retailer appears in all consumers' awareness sets, but the other retailers appear only in some fraction of consumers' awareness sets. As such, notice the prominent retailer's advantage now extends beyond the first search to follow-on searches.

We find that results carry over from the previous model, but with a twist. Previously, search traffic concentration intensified competition, lowering average market prices with the prominent retailer charging a strictly *higher* relative price. Here, the prominent retailer charges (stochastically) *higher* relative prices only in the region where its first-search advantage exceeds a critical threshold. In this region, traffic concentration softens competition, yielding *higher* average market prices and *lower* consumer welfare. In contrast, below the critical threshold, the prominent retailer charges *lower* relative prices, and our previous results hold with traffic concentration intensifying competition, yielding *lower* average market prices and *higher* consumer welfare.

Intuitively, these contingent effects arise from a particular tension in the prominent retailer's potential customer mix. On the one hand, the prominent retailer continues to capture more high-search-cost consumers on account of its first-search advantage. On the other hand, the prominent retailer potentially attracts more low-search-cost consumers as well because of its beyond-first search advantage (recall it is now part of all customers' awareness sets). The relative sizes of these two forces cross at a threshold level of first-search. Above this threshold, the prominent retailer is super-prominent, its potential customer mix includes a higher proportion of high-search-cost individuals compared to its competitors. This less-elastic demand commands higher prices. Conversely, below the threshold, the prominent retailer is moderate-prominent, its potential customer mix includes a relatively higher proportion of low-search-cost individuals, with the more-elastic demand inducing lower relative prices.

Our contingent relative price results rationalize divergent commentaries about online pricing; e.g., Hanbury (2018) crowns Amazon as the cheapest of all leading online retailers, whereas Peterson (2018) notes that Amazon's prices are higher than at Walmart for many popular products. It also focuses policy concerns on this region with super-prominence. To illustrate, in the intra-brand setting, prominence can increase market average prices and hurt consumer welfare in a market characterized by (a) consumers limiting themselves to searching only from their own consideration/ awareness subset of competing retailers (b) a super-prominent retailer that is able to charge higher relative prices than its competitors.

The remainder of the paper is organized as follows. Immediately following, we review the literature selectively. Sections 3 and 4 present our analyses. Section 5 concludes with implications and directions for future search.

Literature Review

A retailer's advantages over competitors has been characterized in several ways, including a first-mover advantage in setting price (Raju and Zhang 2005; Kolay and Shaffer 2013), a cost advantage (Dukes, Gal-Or, and Srinivasan 2006), and the ability to determine assortments first (Dukes, Geylani, and Srinivasan 2009). Vertical advantages include the power to dictate upstream price (e.g., Geylani, Dukes, and Srinivasan 2007).

Search traffic advantage is qualitatively different from all these previous notions, and is uniquely important online. Consider that geographic distances invariably limit the number of effective offline competitors, thus softening intra-brand competition. E-commerce makes geography virtually irrelevant, greatly increasing intra-brand competition on account of the many additional retailers brought within effective reach of a customer. Surprisingly, significant price dispersion persists online for the identical product.

Baye, Morgan, and Scholten (2006) identify two streams of literature that speak to this price dispersion for the same product. One stream emphasizes asymmetries in customer preferences with a "store-loyal" segment of consumers knowing (or caring) about the price only at one retailer; and a "switcher" segment knowing/caring about all retailers' prices. Equilibrium price dispersion is the standard result in this setup (e.g., Varian 1980; Narasimhan 1988; Raju, Srinivasan, and Lal 1990; Baye and Morgan 2001; Chen, Iyer, and Padmanabhan 2002)

Our analyses link to this stream in that we also focus on customer asymmetry, albeit search asymmetry, and is closely related to Koçaş and Kiyak (2006) and Koçaş and Bohlmann (2008), which explores asymmetry in oligopolistic competition settings. We build on these works by characterizing consumers' endogenous search decisions, thus endogenizing the consumer's reservation price. This enables us to unpack the influence of retail price competition on the consumer’s incentive to search, which is the driving force of our pro-competitive effect of search traffic concentration and our curse of prominence results.

The second stream of work on price dispersion identified in Baye, Morgan, and Scholten (2006) is the direct antecedent to our analyses. In Stahl’s (1989) canonical work, symmetric retailers compete to sell a homogenous product to two heterogenous search-cost customer segments (zero and positive search costs respectively). The standard result is price dispersion in mixed strategies (e.g., Stahl 1989; Janssen, Moraga-Gonzalez, and Wildenbeest 2005; Janssen and Shelegia 2015; Jiang, Kumar, and Ratchford 2017). We build on these works by characterizing concentrated search patterns yielding search traffic asymmetry across retailers, which leads to asymmetric price dispersion.

Our research also connects to the broader consumer search literature (e.g., see Stigler 1961; McCall 1979; Weitzman 1979, Diamond 1971; Wolinsky 1986; Stahl 1989; Anderson and Renault 1999; Lal and Sarvary 199; Kuksov 2004; Janssen, Moraga-Gonzalez, and Wildenbeest 2005; Janssen and Shelegia 2015; Jiang, Kumar, and Ratchford 2017; Zhu and Dukes 2017; Ke and Lin 2019; Dukes and Zhu 2019; Zhong 2020; Zou and Jiang 2020).

Within this large and diverse literature, our work is most closely informed by the ordered search stream (e.g., Arbatskaya 2007; Armstrong, Vickers, and Zhou 2009; Wilson 2010; Armstrong and Zhou 2011; Xu, Chen, and Whinston 2011; Astorne-Figaria and Yankelevich 2014; Choi, Dai, and Kim 2018; Petrikaitė 2018; Mamadehussene 2019; Cao and Zhu 2020; Janssen and Ke 2020). Here, instead of randomly sampling from the available options, customers search in a deliberate sequence, visiting certain firms early in the sequence, thus advantaging those firms.

We build on extant ordered search models in two ways. First, extant works focus solely on the advantage of being searched early in the ordered sequence. In contrast, we consider not only a first-search advantage that captures more high-search-cost consumers, but also advantages beyond first-search that attracts visits from more low-search-cost consumers. The resulting tension yields our novel pricing results wherein the prominent retailer's price may be higher or lower than its competitors depending its first search advantage magnitude.

Second, and perhaps, the most significant, our work focuses on an intra-brand setting, whereas much of the extant ordered search work concerns inter-brand competition (i.e., heterogenous products). For instance, contrast our work with Armstrong, Vickers, and Zhou (2009) which represents the closest extant work on ordered search. Their customers are heterogenous in their product valuations, but their model abstracts away from search cost differences. Their takeaway is that the prominent firm (i.e., the firm searched first by all consumers) always charges a lower relative price to dissuade consumers from searching for a better fitting product, so searches beyond the prominent firm are those customers with lower match values to it. Consumers are being sorted based on their matching value, which increases effective differentiation across sellers. As such, more prominence always decreases price competition and always increases a firm's profit.

In contrast, in our intra-brand setting, the prominent retailer can charge a higher relative price to leverage its advantage of being searched first. Different from Armstrong, Vickers, and Zhou (2009), our consumers are sorted based on their search costs. Consumers who search beyond the prominent retailer are those with low search cost, which intensifies the competition between the fringe retailers. In turn, this lowers the reservation price at the prominent retailer. In sum, prominence can intensify price competition and more traffic can reduce a retailer's profit.

Prominence under Full Awareness

Three retailers carry an identical product, with wholesale prices normalized to zero. A unit mass of consumers exists, and each consumer exhibits unitary demand. Her utility from buying the product is $U = v - p$, where v is product value and p is price.[4] She knows v and searches retailers sequentially for prices. For now, we assume that all consumers are aware of all retailers. She will choose to visit an additional retailer only if her incremental search benefit exceeds her incremental search cost. Finally, we assume perfect recall so the consumer can purchase from any retailer she has previously visited.

As is commonly assumed in search models (e.g., Stahl 1989; Kuksov 2004), all consumers have a "free" first search.[5] However, unlike prior work, we assume that each consumer has a free first search at only one retailer; i.e., her "default" retailer.[6] Behaviorally, we attribute this zero search cost to her history and familiarity with her default retailer.[7] Also, we assume that consumers have heterogeneous "default" retailers.[8]

There are two discrete consumer segments; the "shopper" segment of size $\mu \in (0,1)$ incurs zero cost to search, while the "non-shopper" segment of size $(1 - \mu)$ incurs positive cost c to undertake each additional search.

Denote the prominent retailer and two symmetric fringe retailer types as d and f respectively. Denote α_i as the fraction of consumers who start their search at retailer i ($i = d, f_1, f_2$), $\sum_i \alpha_i = 1$. The prominent retailer has the highest first search level, so $\alpha_d \in (\frac{1}{3}, 1]$ and $\alpha_d > \frac{1}{3} > \alpha_f$ given symmetric fringe.

At the first stage of the game, retailers simultaneously set prices. Retailers cannot identify a consumer's search cost, so price discrimination is ruled out. At the second stage, the consumer searches for prices and purchases the product.

The equilibrium concept is Perfect Bayesian Equilibrium (PBE). Under PBE, retailers maximize profit π and set prices simultaneously conditional on their first search level (α_i), and their expectations about consumer behavior consistent with equilibrium search strategy. The consumer maximizes her utility and makes search decisions conditional on her belief about retailers' pricing strategies consistent with equilibrium strategy. We are interested in the equilibrium where the fringe retailers play a symmetric strategy.

Consumer Search and Retailer Price Strategy

The shopper segment customer searches exhaustively over all retailers and purchases from the lowest price retailer. We can think of this customer as accessing all price quotes from a price comparison or clearing-house website. The non-shopper segment customer stops searching when her expected search benefit is not higher than her cost. Suppose she observes a price z at retailer i, then her search benefit is finding a price p<z. Given price distribution $F_j(p)$ and its lower bound $\underline{p}_j$ from the best alternative retailer j, her expected search benefit after finding a price z at retailer i is

$$EB(z) \equiv \int_{\underline{p}_j}^{z} F_j(p)dp. \tag{1}$$

$EB(z)$ monotonically increases with z. Put differently, a non-shopper's search benefit is higher if she encounters higher prices at retailer i.

Non-shoppers stop searching if one retailer's price is sufficiently low. The highest price that stops the search at retailer i is r_i, which equalizes the search benefit to cost:

$$EB(r_i) = c. \tag{2}$$

Denote r_i as the endogenized reservation price at retailer i. If $p \leq r_i$, she stops searching and purchases from retailer i. Otherwise, she continues to search.

Consider the prominent retailer. If the customer is dissatisfied with the price offer, her alternative is to visit a fringe retailer. Thus, her endogenous reservation price at the prominent retailer r_d satisfies $c = \int_{\underline{p}_f}^{r_d} F_f(p)dp$.

In contrast, at a fringe retailer, if she is dissatisfied with the price offer, her alternative is to visit either the prominent retailer or else the other fringe retailer, whichever has a higher search benefit. Thus, her endogenous reservation price at the fringe retailer r_f satisfies $c = \max\left\{\int_{\underline{p}_d}^{r_f} F_p(p)dp, \int_{\underline{p}_f}^{r_f} F_f(p)dp\right\}$.

Below, we present our analysis and results. All proofs are in the Appendix.

Lemma 1. *There is no pure strategy equilibrium.*

Intuitively, each retailer can be searched by both shoppers and non-shoppers. Thus, it faces a tradeoff between these segments. On the one hand, a retailer has an incentive to charge a high price to extract surplus from its first-search non-shoppers. On the other hand, it is tempted to charge a low price to compete for shoppers who visit more than one retailer. Given these conflicting incentives, retailers would deviate from any single price. Consequently, if the equilibrium exists, some, if not all, retailers must play mixed strategies.

Retailer i charges prices between $p \in [\underline{p}_i, \overline{p}_i]$ with CDF $F_i(p)$, where $\underline{p}_i$ and $\overline{p}_i$ are the lower and upper boundary of retailer i's price support. Under any mixed strategy equilibrium, the retailer's expected profit must be constant at any $p \in [\underline{p}_i, \overline{p}_i]$.

Lemma 2. *Under any mixed strategy equilibrium, the price support of prominent retailer is convex for* $p \le r_f$, *while the price support of fringe retailer is convex for* $p \le r_d$.

Intuitively, if there is a "hole" in the price support of prominent retailer for $p \le r_f$, either prominent or fringe retailer would not have constant profits within its price support. For example, the competing fringe could have a higher profit at the upper than the lower boundary of the "hole". This would violate the definition of mixed strategy equilibrium.

Now consider $\overline{p}_i$, the upper boundary of the price distribution. First, no retailer would price higher than consumer's valuation to the product, so we must have $\overline{p}_i \leq v$.[9] Second, consider the relationship between reservation price for search and upper boundary. The Lemma below summarizes this relationship.

Lemma 3. *Under any mixed strategy equilibrium,* $\overline{p}_i \leq r_i$.

Under any mixed strategy equilibrium, all retailers price no higher than their endogenized reservation prices to lock in their first-search non-shoppers. Otherwise, it would either lead to a decrease in its profit or else violate Lemma 2. This Lemma implies that the non-shopper segment customers purchase at the first retailer they visit under any mixed strategy equilibrium.

Mixed Strategy Equilibrium

All shopper segment customers search exhaustively, visiting all three retailers and buy at the lowest price, while all non-shoppers search and purchase from the first retailer they visit. The equilibrium profit of retailer i for $p \in [\underline{p}_i, \overline{p}_i]$ under the mixed strategy is as follows:

$$E\pi_d = \{(1-\mu)\alpha_d + \mu[1 - F_f(p)]^2\}p, \tag{3.1}$$

$$E\pi_f = \{(1-\mu)\alpha_f + \mu[1 - F_d(p)][1 - F_f(p)]\}p. \tag{3.2}$$

$(1-\mu)\alpha_d$ and $(1-\mu)\alpha_f = \frac{(1-\mu)(1-\alpha_d)}{2}$ represent the non-shoppers who start their search at the prominent and fringe retailer respectively, and μ stands for the shoppers who search exhaustively. The probabilities of acquiring shoppers are $[1 - F_f(p)]^2$ f and$[1 - F_d(p)][1 - F_f(p)]$ for the prominent and fringe retailers respectively.

Given the retailer's expected profit function and consumer's search strategy, we solve for the equilibrium price distribution and reservation price. Proposition 1 summarizes the equilibrium outcome.

Proposition 1 (Equilibrium price)

Under full awareness, given any $\alpha_d > \frac{1}{3}$, *the prominent retailer charges a higher price than fringe retailers. Specifically,* $p_d = r$ *with probability one, while the fringe retailers randomize prices between* $p \in [\underline{p}, \overline{p}]$ *following* $F_f(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{2\mu}(\frac{r}{p} - 1)$, *where* $\overline{p} = r$, $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+2\mu} r$, *and* $r = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}$.

Intuitively, consider that any retailer's customer mix potentially includes non-shoppers and shoppers who visit it. As shown previously, in equilibrium, shoppers search all three retailers while non-shoppers only search their default retailers. Retailer *i*'s mix of non-shoppers to shoppers (call it non-shopper ratio) is given as $\frac{\alpha_i(1-\mu)}{\mu}$. Recall non-shoppers do not compare prices across retailers, so they are effectively less price sensitive. As such, a higher non-shopper ratio implies a less price-sensitive customer mix for this retailer.

Due to its first-search advantage, the prominent retailer is searched by more non-shoppers than any individual fringe, while shoppers visit everyone, so the prominent retailer always has a higher non-shopper ratio than any individual fringe. Its faces a less-elastic demand, enabling a higher price.

Fringe retailers play the mixed strategy due to the tradeoff between shoppers and non-shoppers. However, such a tradeoff is muted for the prominent retailer. The price competition between fringe retailers makes it unattractive for the prominent retailer to poach shoppers. Therefore, the prominent retailer only acquires the non-shoppers who start their search there.

This equilibrium outcome is reminiscent of extant asymmetric oligopoly models. For example, Koçaş and Kiyak (2006) finds that only two retailers compete for switchers in an oligopoly market. Anderson, Baik, and Larson (2015) reports that only top two firms with the highest quality levels advertise in an asymmetric oligopoly market.[10]

Proposition 2 (Pro-competitive effect of search-traffic concentration)

Under full awareness, an increase in α_d *always lowers average prices at both types of retailers.*

Intuitively, shoppers search all three retailers, so a fringe retailer needs to undercut the price of the prominent retailer as well as the other fringe to acquire shoppers. When first-search traffic is increasingly more concentrated at the prominent retailer, this leaves fewer non-shoppers starting at either fringe retailer, leaving them to compete for the price-sensitive shoppers. This intensifies the inter-fringe competition. What is more, lower fringe prices make an additional search even more appealing to the non-shoppers who start at the prominent retailer. Thus, they would cease searching at a lower reservation price. In order to keep these customers, the prominent retailer has to lower its price as well.

Proposition 3 (The effect of search-traffic concentration on consumer welfare)

Under full awareness, an increase in α_d *increases consumer welfare.*

Absent product heterogeneity, price is the key factor that determines consumer welfare. Since higher search traffic concentration lower average prices of all retailers, this improves consumer welfare. We caution this benign welfare implication of search traffic concentration exists in a homogenous product setting when product valuation is sufficiently high compared to the consumer search cost.

Proposition 4 (The curse of prominence)

Under full awareness, when the size of shopper segment (μ) *is small and* $\alpha_d > \frac{1}{2}$, *an increase in* α_d *lowers prominent retailer profit.*

Intuitively, this "curse of prominence" arises from the tradeoff between *demand enhancement* and *price competition* effects. On the one hand, more first-search traffic increases the number of non-shoppers that the retailer can capture; on the other hand, it dampens the retailer's margin by its influence on the overall competitive landscape.

The curse is realized when the size of non-shopper segment and the prominent retailer's first search advantage are sufficiently large. The margin decline due to the price competition effect hurts the prominent retailer more when it has a larger demand per se. Recall the prominent retailer has a larger demand when the size of the non-shopper segment and its first-search advantage are sufficiently large, because it only captures non-shoppers who visit it first. Therefore, when more non-shoppers start their search at the prominent retailer, the negative price competition effect dominates the positive demand enhancement effect. In such a case, more first-search traffic reduces the prominent retailer's profit.

Corollary 1 (The blessing of prominence)

Under full awareness, when the size of shopper segment (μ) *is large or* $\alpha_d \leq \frac{1}{2}$, *an increase in* α_d *enhances prominent retailer profit.*

Prominence is not always a curse to the prominent retailer. When the prominent retailer does not have a sufficiently high demand, the positive demand enhancement effect of prominence dominates the negative price competition effect. In such cases, more first-search traffic increases the prominent retailer's profit even though it lowers its margin per unit.

Prominence under Limited Awareness

Earlier, we remarked that consumers typically consider only a limited subset of online sellers. Here, we incorporate limited awareness into our analysis as follows. Denote β_i as the fraction of consumers who are aware of retailer i. Each consumer is aware of only two out of the three retailers, $\sum_i \beta_i = 2$. All consumers consider only those retailers included in their particular awareness sets, so even a zero-search cost shopper does not visit retailers not included in her particular awareness set.

Consistent with empirical reports about asymmetric awareness (e.g., 92% of US consumers visited Amazon, NPR 2018), we assume the prominent retailer appears in all customers' awareness sets, while each of the two fringe retailers appears only in half of the awareness sets, so we have $\beta_d = 1 > \beta_f = \frac{1}{2}$. Notice that the prominent retailer is more likely to be visited even by those consumers who start their search elsewhere. This comports with empirical reports like Bloomberg (2016) where 90% of consumers will check Amazon even if they searched another retailer first.

Our limited awareness model serves three purposes. First, it complements our previous characterization of prominence as first-search advantage to incorporate the additional aspects described immediately above. Second, it addresses an anecdotal puzzle left unaddressed in the earlier model. Recall the strictly higher equilibrium price of the prominent retailer relative to its competitors. In fact, industry practice suggests higher (e.g., Peterson 2018) as well as lower (e.g., Hanbury 2018) relative prices at prominent retailers. Third, it establishes the boundaries of our earlier results, particularly the pro-competitive consequences of search traffic concentration and the curse of prominence. Do they carry over unchanged, or are they more circumscribed with limited awareness?

Shopper segment customers search only those two retailers (the prominent and one fringe retailer) that appear in their own awareness set. Second, non-shopper segment customers who are dis-satisfied with the price found at the prominent retailer have the option to visit a fringe retailer, and vice versa. As such, the endogenized reservation prices r_d and r_f satisfy

$$c = \int_{\underline{p}_f}^{r_d} F_f(p)dp = \int_{\underline{p}_d}^{r_f} F_d(p)dp. \tag{4}$$

We characterize retailer i's profit for $p \in [\underline{p}_i, \overline{p}_i]$ given the other's pricing strategy as:

$$E\pi_d = \{(1-\mu)\alpha_d + \beta_d\mu[1 - F_f(p)]\}p, \tag{5.1}$$

$$E\pi_f = \{(1-\mu)\alpha_f + \beta_f\mu[1 - F_d(p)]\}p. \tag{5.2}$$

$(1-\mu)\alpha_d$ and $(1-\mu)\alpha_f = \frac{(1-\mu)(1-\alpha_d)}{2}$ represent the non-shoppers who start their searches with the prominent and fringe retailer respectively. Moreover, $\beta_d\mu = \mu$ and $\beta_f\mu = \frac{\mu}{2}$ represent shoppers who exhaustively search each retailer within their own awareness sets. Different from the full awareness case, the prominent retailer now is also searched by more shoppers than the fringe because it appears in more awareness sets, $\beta_d > \beta_f$. The probability of acquiring shoppers is $[1 - F_f(p)]$ and $[1 - F_d(p)]$ for the prominent and fringe retailer respectively. Proposition 5 summarizes the equilibrium price pattern depicted in Figure 1.

Proposition 5. (Equilibrium price)

(1) When $\alpha_d \geq \alpha^* = \frac{1}{2}$, *the prominent retailer charges higher relative prices stochastically.*

Specifically, $F_d(p) = \frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}] \leq F_f(p) = 1 - \frac{(1-\mu)\alpha_d}{\mu}(\frac{r_d}{p} - 1)$ *for* *any* $p \in [\underline{p}, r_d)$*, where* $\underline{p} = \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_p$ *and* $r_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]}$. $F_d(p)$ *has a mass point at* r_d *with probability* $\frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}$.

(2) When $\alpha_d < \alpha^*$*, the prominent retailer charges lower relative prices stochastically. Specifically,* $F_d(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}\left(\frac{r_f}{p} - 1\right) > F_f(p) = \frac{(1-\mu)\alpha_d+\mu}{\mu}\left[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}\right]$ *for any* $p \in [\underline{p}, r_f)$*, where* $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu} r_f$ *and* $r_f = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}$. $F_f(p)$ *has a mass point at* r_f *with probability* $\frac{(1-\mu)\alpha_d+\mu}{(1-\mu)(1-\alpha_d)+\mu}$.

The right-hand panel of Figure 1 shows that when the first-search advantage is beyond the critical threshold, $\alpha_d \geq \alpha^*$ (we denote the prominent retailer to be super-prominent in such cases), the prominent retailer's equilibrium price first-order stochastically dominates the fringe retailer's, $F_d(p) \leq F_f(p)$ for any p within the price support. The left-hand panel of Figure 1 describes the opposite pattern when the first-search advantage is below the critical threshold.

<Insert Figure 1 Here>

Intuitively, sufficiently high first-search levels enable the prominent retailer to capture more non-shoppers. On the other hand, its high awareness also allows it to be searched by more shoppers. The former force prevails beyond the critical threshold level where the prominent retailer has a relatively higher non-shopper ratio, $\frac{(1-\mu)\alpha_d}{\mu} \geq \frac{(1-\mu)(1-\alpha_d)}{\mu}$, so it faces a less-elastic demand. This leads to higher prices at the prominent retailer.

The latter force dominates below the critical threshold where its higher awareness rate allows the prominent retailer to capture more shoppers relative to non-shoppers compared with its competitors, resulting a lower non-shopper ratio. Facing this more-elastic demand, the prominent retailer charges lower prices compared with its competitors.

The prominent retailer's profit is provably higher than the fringe retailer's profits for any first-search advantage level (α_d). Search traffic dominance gives the prominent retailer the opportunity to earn a higher profit than its competitors whether it leverages its advantage through higher or lower prices versus its competitors.

Proposition 6 (Average price, consumer welfare, and profit)

(1) When $\alpha_d \geq \alpha^*$*, an increase in* α_d *increases the average prices of all retailers, lowering consumer welfare. When* $\alpha_d < \alpha^*$*, an increase in* α_d *lowers the average prices of all retailers, raising consumer welfare.*

(2) The prominent retailer's profit decreases with α_d *when* $\alpha_d < \alpha^*$ *and* $\mu > \mu_1(\alpha_d) = \sqrt{\frac{1}{2} + \frac{{\alpha_d}^2}{16(1-\alpha_d)^2}} - \frac{\alpha_d}{4(1-\alpha_d)}$.

More concentrated first-search exhibits a non-monotonic impact on price competition and thus consumer welfare. Here, shoppers search only two retailers (the prominent and one fringe), so one fringe only needs to undercut the price of the prominent retailer to acquire shoppers. In this case, the price competition level is determined by the difference in customer mix between the prominent and fringe retailer. Price competition is intensified when the two retailers share more similar customers, which are measured by the non-shopper ratios.

Below the critical threshold, the prominent retailer has a lower non-shopper ratio than the fringe retailers. Increases in α_d increases this ratio at the prominent retailer, but decreases it at the fringe retailer, bringing their customer mixes closer together. This intensifies price competition and lowers averages prices of all retailers, benefiting consumers. When $\alpha_d = \alpha^*$, all retailers share the same non-shopper ratio. An increase in α_d beyond α^* (or when the retailer is super-prominent) further differentiates the retailers' customer mixes, softens price competition, and thus increases average prices for all retailers. Consequently, consumers can be hurt from search traffic concentration when $\alpha_d \geq \alpha^*$.

Notice the curse of prominence is realized at mild first-search advantage levels and when there are more shoppers; this is different from the full awareness case. The intuition is as follows. First, the negative price competition effect only exists at mild first-search levels. In addition, the prominent retailer has a larger demand when the market has a higher proportion of shoppers, because it has more shoppers relative to non-shoppers. Recall that the decline in margin due to price competition effect hurts the prominent retailer more if it has a larger demand. As such, the negative price competition effect dominates the demand enhancement effect when the prominent retailer has a mild first-search levels and the market contains a high proportion of shoppers.

General Discussion

Contributions to Scholarship

We examine the market outcome when search traffic concentrated at a prominent retailer selling an identical product in competition with other retailers. Our results rationalize divergent commentaries and observations about prominent retailers' prices. For example, we show when the prominent retailer charges relatively higher (e.g., see Peterson 2018) and when it charges lower prices than its competitors (e.g., see Hanbury 2018). We also show that prominence can increase or decrease market average prices with corresponding decreases and increases in consumer welfare. Counter-intuitively, we find that prominence can be privately beneficial (blessing) or privately harmful (curse) depending on the strength of positive demand enhancement versus negative price competition effect.

Our study broadens our understanding of asymmetries in retail competition beyond those emphasized in extant work (e.g., Raju and Zhang 2005; Dukes, Gal-Or, and Srinivasan 2006; Geylani, Dukes, and Srinivasan, 2007; Dukes, Geylani, and Srinivasan 2009). Amongst the growing ordered search literature (e.g., Armstrong, Vickers, and Zhou 2009; Xu, Chen, and Whinston 2011; Choi, Dai, and Kim 2018), this work is most closely related to the Armstrong, Vickers, and Zhou (2009). We highlight the following differences to this seminal work.

Regarding model setup, our consumers incur heterogeneous search costs (but exhibit homogenous product valuation), whereas Armstrong, Vickers, and Zhou (2009) study consumers with heterogeneous product valuations (but exhibit homogenous search cost). These differences yield divergent takeaways. First, in Armstrong, Vickers, and Zhou (2009), the prominent firm always charges a lower relative price to constrain consumers from searching further for a better fitting product elsewhere. However, in our intra-brand setting, the homogenous product but heterogenous search costs allow the prominent retailer to leverage its advantage of being searched first with a higher relative price. Second, prominence in Armstrong, Vickers, and Zhou (2009) sorts consumers based on their product fit, thus further differentiating sellers. Consequently, prominence always reduces price competition, hurts consumers, and benefits firms. Whereas, in our models, prominence sorts consumers based on their search costs. Being more prominent leaves fewer high-search-cost consumers to fringe competitors, which intensifies competition between them and can lower the reservation price at this prominent retailer. As such, prominence can encourage competition, benefitting consumers, but possibly hurting retailer profits.

To conclude, our results show that one needs to consider the setting carefully to understand the effects of prominence. Our model provides insights in intra-brand settings when consumer search cost heterogeneity is important, while Armstrong, Vickers, and Zhou (2009) provides insights into inter-brand settings where product fit heterogeneity is important. Prominence can help or hurt the firm's profit when search cost heterogeneity is the key driver of consumer choice, but prominence improves profits if the heterogeneity is in match quality rather than search costs

Our work also yields empirically refutable hypotheses as avenues for future research. Higher versus lower relative prices at the prominent retailer are shown to the contingent on the firm's level of first search advantage. Above a crossover point, the prominent retailer charges relatively higher prices (stochastically), and vice versa. Koçaş and Bohlman (2008) develop the methodology for assessing these stochastic dominance predictions.

Implications for Policy

Recent regulatory investigations (e.g., Competition and Markets Authority 2020) as well as legislative inquiries (e.g., Subcommittee on Antitrust 2020) point to asymmetries and dominance along various dimensions including search traffic concentration as problematic characteristics of online markets. However, antitrust scholars (e.g., Khan 2019) contend that our contemporary policy framework predicated on showing that a suspect pattern or practice is indeed harmful to consumer welfare is limited in its ability to fashion policy guidance in online markets. The novel aspects of these markets such as search traffic patterns, and "free" products such as search engines and social media are not well understood in their effects on consumer welfare (e.g., Competition and Markets Authority 2020).

Our welfare results improve our ability to frame these policy concerns and to fashion remedies under intra-brand setting. First of all, search concentration is neither an unalloyed good or bad thing with respect to consumer welfare, thus suggesting a rule-of-reason approach for assessing traffic concentration concerns. Our specific results help to locate those particularly fragile online markets where consumer welfare is likely to be detrimentally affected by prominence. Markets characterized by (a) a super-prominent retailer that is able to charge higher relative prices than its competitors and (b) consumers limiting themselves to searching only from their own consideration/ awareness subset of competing retailers are targets for policy remedies. Absent all these characteristics, our analyses suggest search concentration principally shifts profits amongst competitors, but is benign from a welfare standpoint.

In addition to targeting regulatory "sticks" at fragile markets, it is useful to identify ameliorative "carrots"; a natural direction here is to ameliorate search cost burdens. Nevertheless, a long-standing literature in Marketing and elsewhere contends that ameliorative initiatives and laws (e.g., unit pricing label regulations) that seek to improve cost of search are often under-utilized (e.g., Isakson and Maurizi 1973) and are thus inefficient policies (e.g, Bergen et al. 2008). Our work shows that the reach of these initiatives is likely under-estimated. The existence of an even small group of shoppers, who access all retailers, can tame the possible downside of prominence on consumer welfare and benefit all consumers. That is to say, expanding even a small group of price-comparing shoppers can discipline the market and yields positive externality to all consumers. As such, policy-makers can encourage price comparison behavior by educating consumers, or provisioning price transparency even if most consumers choose not to use these tools.

Takeaways for Retailers

The first takeaway from our analyses is that retailers must attend to the composition of these customer mix in order to adapt to their search traffic. What percentage of their consumers started at their site? What percentage of consumers continue to search after visiting them? How many retailers do consumers consider searching before making a purchase? Albeit at some expense, all these elements are discoverable. For instance, one can survey customers to assess top-of-mind awareness and order of recall as surrogates for order of search (e.g., Amaldoss and He 2013). Online tracking tools are also available to track journeys across sites.

Our work also highlights divergent customer postures at prominent versus fringe retailers. Recall that fringe retailer in general compete more for low-search-cost consumers, who are more price-sensitive. As such, a transactional marketing approach is indicated for these retailers. Instead of trying to retain their first-stop customers, they need to adapt transactionally using high-low pricing, time-limited deals, flash sales, etc. all intended to induce customers who started elsewhere to visit them as a further stop.

In direct contrast, recall our prominent retailer's profits in general pivoted on retaining their high-search cost first-stop customers. We know these customers will stop their search at any price up to the level of their endogenous reservation price, r. Inspecting the expression for r (Proposition 1), we see that r increases directly with search cost, c. One way to understand a search cost is the anticipated expenditure of incremental time, and/or money expended by searching further. As such, relationship marketing tools building on tactics such as loyalty programs, customized apps and clientization is indicated here.

Limitation and Future Research

To close, consider the limitations and boundaries of our work. First, we caution our welfare-improvement results are relevant to an intra-brand setting. Second, our operationalization of prominence involves a free search only at her default retailer. The roots of this match between a customer and her default retailer are abstracted away; we speculate that reputation, advertising, service quality and variety are all likely primitives, but these matters are left to future work. Third, we abstracted away other differentiation across retailers to focus on search traffic asymmetries. Additional inter-retailer differentiation along non-core product dimensions like shipping costs etc. is left for future work. Finally, we abstract away other channel interactions, particularly interactions between the product manufacturer and the retailers by normalizing to a constant, exogenous wholesale price. It might be fruitful to explore the vertical channel implications of a retailer's search traffic dominance. Specifically, how does the existence of a prominent retailer influence the manufacturer's price and product choice? How should a manufacturer govern the channel when the search traffic is concentrated? How should retailers make price and assortment decision with the arise of a prominent retailer? We trust future research will advance insights into these questions.

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Footnote

[1] The dominant online retailer varies across products (e.g. chewy.com leads traffic volume in pet food and supply (https://www.similarweb.com/website/chewy.com/?competitors=petco.com/) and lowes.com leads in home improvement and maintenance (https://www.similarweb.com/website/lowes.com/?competitors=goodhousekeeping.com/). Accessed April 2021.

[2] Intra-brand competition is described by Coughlan, Anderson, Stern and El-Ansary (2001, p.349) as follows: "…competition among wholesalers or retailers of the same brand…… product."

[3] This research focus on teasing out the effect of prominence on competition and consumer welfare under intra-brand setting. However, amazon's advantage over other retailers may extend beyond search prominence, which includes factors such as cost efficiency, network externalities, and is beyond the scope of interest in this paper. Consequently, readers need to be cautious in interpreting our results beyond the framework of prominence in intra-brand setting.

[4] We assume v to be sufficiently large compared to search cost c to focus on the impact of consumer search on market outcome.

[5] See for example, Kuksov (2004), "The assumption that a consumer can obtain one price quote at no cost is a technical assumption that ensures that the market exists." (page 492)

[6] The free search at this default retailer explains why this consumer starts her search with it. In Proofs of Proposition 1 and 5, we show that the zero-search-cost assumption guarantees that the consumer's expected utility from searching her default is weakly higher than her utility from searching a non-default retailer.

[7] For example, if a consumer is familiar with one retailer, she might know the feature of this retailer's website well, which lowers her cost to find price quote from it. Also, this "default" retailer might keep a record of the consumer's payment and mailing information, which reduces her cost of interacting with this retailer.

[8] Heterogeneity in "default" search is similar to heterogeneous loyal segments in Narasimhan (1988) and Chen, Narasimhan, and Zhang (2001), except that the consumer's reservation price is endogenized through their search decisions in this model.

[9] In the main model, we consider the case that $v \geq r_i(c)$ for any i since we are interested in the implication of search on firm's pricing. In the Web Appendix, Section A, we consider the case that $v < r_i(\mathrm{c})$, where retailer's price is capped by product valuation.

[10] The pure strategy of the prominent retailer is driven by the nature of oligopolistic competition. In the web appendix, we show that the prominent retailer utilizes a pure strategy even when fringes have asymmetric first-search share.

Appendix

Proof of Lemma 1

In this section, we show that there is unilateral deviation from any pure strategy equilibrium. Suppose that there exists a pure strategy equilibrium, and without loss of generality, let's assume the equilibrium price to be p_d, p_f and $p_d < p_f$. Note that the same logic holds for $p_d \geq p_f$.

First, we want to show that $p_f - p_d < c$ in equilibrium. If $p_f - p_d \geq c$, then non-shoppers who visit the fringe retailer first would make an addition search at the prominent one. Since $p_d < p_f$, all consumers would purchase from the prominent retailer. Consequently, the fringe would have a zero demand and thus a zero profit, which is lower than the positive profit for the fringe under $p_f < p_d + c$. This proves $p_f - p_d < c$.

Next, we show that no equilibrium exists for $p_f - p_d < c$. In this case, non-shoppers buy from the first retailer they visit, while all shoppers purchase from the prominent one because its price is lower. If this is the equilibrium, there must exist $\epsilon \to 0$ such that $p_f - p_d < \epsilon$. Otherwise, the difference between p_f and p_p would be large enough to attract the prominent retailer to deviate to $p_d' \in (p_d, p_f)$, under which it has a higher margin without losing any demand. However, if there exists $\epsilon \to 0$ such that $p_f - p_d < \epsilon$, one fringe would deviate to a price slightly lower than p_d, under which it has a discontinuous demand increase by capturing all shoppers. Thus, no equilibrium exists for $p_f - p_d < c$.

The above shows that no pure strategy price equilibrium exists for $p_d < p_f$. Q.E.D.

Proof of Lemma 2

We prove lemma 2 by contradiction. We will show that if there exists a "hole" at the price support of fringe retailer when $p \leq r_d$, either the prominent and fringe retailer would have inconstant profits within the price support. This contradicts the definition of mixed strategy equilibrium. (Note that the same logic holds for the price support of prominent retailer when $p \leq r_f$ as well.)

Consider the fringe retailer competes for shopper with another symmetric fringe and the prominent retailer. If there exists a "hole" (a, b) at the price support of the fringe retailer for $p \leq r_d$, it implies that this fringe puts zero probability for $p \in (a, b)$. The hole (a, b) suggests $F_f(p') = F_f(a)$ for any $p' \in [a, b]$. Suppose $(\underline{p}_d, \overline{p}_d) \cap (a, b) = (a', b')$, where $a \leq a' \leq b' \leq b$, we must have $F_f(a) = F_f(a') = F_f(b') = F_f(b)$. Considering the relationship between the price support of the fringe and prominent retailer, there can be different cases regarding to (a', b'). We consider all possible cases as follows.

First, let's consider the case that $(a', b') \neq \emptyset$. Because $b' < b < r_d$, non-shoppers who start with the prominent retailer stop searching and purchase from it at both $p = a'$ or $p = b'$. However, this leads to a higher profit for the prominent retailer under a' than b':

$$E\pi_d(a') = \{(1-\mu)\alpha_d + \mu\beta_d[1 - F_f(a')]^2\}a'$$
$$< E\pi_d(b') = \{(1-\mu)\alpha_d + \mu\beta_d[1 - F_f(b')]^2\}b',$$

because $a' < b'$ and $F_f(a') = F_f(b')$. The inconstant profit within the price support contradicts to the definition of mixed strategy equilibrium.

Second, if $(a', b') = \emptyset$, it implies either $a < b < \underline{p}_d$ or $b > a > \overline{p}_d$. In both cases, for any $p' \in [a, b]$, we have $F_d(a) = F_d(p')$. In this case, there can be three possible scenarios (1) $a < r_f$, (2) $a = r_f$, and (3) $a > r_f$. Let's consider all three, respectively.

(1) $a < r_f$

In this case, at either a or $\min\{b, r_f\}$, the non-shoppers at the fringe retailer stop search and buy from this fringe. We know that for any $p' \in [a, b]$, $F_d(p') = F_d(a)$ and $F_f(p') = F_f(a)$, then we must have $F_f(a) = F_f(\min\{b, r_f\})$ and $F_d(a) = F_d(\min\{b, r_f\})$. This shows that the fringe retailer has lower expected profit at a than $\min\{b, r_f\}$ given

$$E\pi_f(a) = \{(1-\mu)\alpha_f + \mu\beta_d[1-F_f(a)][1-F_d(a)]\}a$$

$$< E\pi_d(\min\{b, r_d\}) = \{(1-\mu)\alpha_d + \mu\beta_d[1-F_f(\min\{b, r_d\})][1-F_d(\min\{b, r_d\})]\}\min\{b, r_d\}.$$

This contradicts the definition of mixed strategy equilibrium.

(2) $a > r_f$

In this case, at either a or b, non-shoppers at the fringe retailer continue to search. Without loss of generalizability, suppose these non-shoppers finds a lower-than-reservation price at the other fringe. In such a case, the fringe will have a strictly lower profit at a than b:

$$E\pi_f(a) = \{(1-\mu)\alpha_f[1-F_f(a)] + \mu\beta_f[1-F_f(a)][1-F_d(a)]\}a$$

$$< E\pi_f(b) = \{(1-\mu)\alpha_f[1-F_f(b)] + \mu\beta_f[1-F_f(b)][1-F_d(b)]\}b,$$

because $a < b$, $F_d(a) = F_d(b)$, and $F_f(a) = F_f(b)$. This violates the definition of mixed strategy equilibrium.

(3) $a = r_f$

In this case, we can find a $p' = a + \epsilon$, where $\epsilon > 0$. Under both $a + \epsilon$ and b, the non-shoppers at the fringe would continue their searches. Following the proof from case (2), we can show that the fringe retailer has a strictly higher profit under b than $a + \epsilon$. This contradicts the definition of the mixed strategy equilibrium.

The above proof also holds for the prominent retailer's price support when $p < r_f$. Together, we can prove that there must exist no "hole" for the price support of prominent retailer if $p \leq r_f$ and for the fringe retailer if $p < r_d$. Q.E.D.

Proof of Lemma 3

We are interested in an equilibrium that two fringe retailers play an identical strategy. We want to show $\overline{p}_f \le r_f$ and $\overline{p}_d \le r_d$. Without loss of generalizability, we will show that $\overline{p}_f \le r_f$ and $\overline{p}_d \le r_d$ when $r_f \le r_d$. The same logic goes through the case that $r_f > r_d$ as well.

First, we show $\overline{p}_d \le r_d$ as follows. If the prominent retailer charges $p > r_d$, it has a zero demand, because both non-shoppers and shoppers can find a lower price from the fringe, whose price is $p \le r_f < r_d$. Thus, the prominent retailer has a zero profit when charging $p > r_d$. However, this is strictly lower than its positive profit under $p = r_d$. This proves $\overline{p}_d \le r_d$.

Second, we show $\overline{p}_f \le r_f$ by contradiction. Suppose a fringe retailer charges $p = r_f + \epsilon$, where $\epsilon > 0$. In this case, non-shoppers who start with the fringe would keep searching the prominent one. There can only be two possible cases: (1) $\overline{p}_d \le r_f$ or (2) $\overline{p}_d > r_f$. Note that the proofs vary in two cases. If $\overline{p}_d \le r_f$, the proof is similar to Stahl (1989), the fringe retailer has a strictly lower profit under $p = r_f + \epsilon$ than r_f. However, if $\overline{p}_d > r_f$ we will show that $p = r_f + \epsilon$ would violate lemma 2.

Case 1. $\overline{p}_d \le r_f$

If $\overline{p}_d \le r_f$, all consumers could find a lower price from the prominent retailer with certainty. Thus, this fringe has a zero profit at $r_f + \epsilon$. However, it is strictly lower than the positive profit at r_f. Thus, $\overline{p}_f \le r_f$.

Case 2. $\overline{p}_d > r_f$

If $\overline{p}_d > r_f$, non-shoppers find a lower price at the prominent retailer with probability. Therefore, they purchase from the retailer with a lower price. This gives us fringe's profit at $r_f + \epsilon$ as

$$E[\pi_f(r_f+\epsilon)] = (r_f+\epsilon)[(1-\mu)\alpha_f+\mu\beta_f][1-F_d(r_f+\epsilon)].$$

Since $F_d(p)$ is non-decreasing, we have

$$E[\pi_f(r_f+\epsilon)] \leq (r_f+\epsilon)[(1-\mu)\alpha_f+\mu\beta_f][1-F_d(r_f)].$$

Combining it with $E[\pi_f(r_f)] = r_f\{(1-\mu)\alpha_f+\mu\beta_f[1-F_d(r_f)]\}$, we can have $E[\pi_f(r_f+\epsilon)] < E[\pi_f(r_f)]$ when $\epsilon < \epsilon_1 = \frac{(1-\mu)\alpha_f F_d(r_f)}{[(1-\mu)\alpha_f+\mu\beta_f][1-F_d(r_f)]} r_f$. This suggests that $E[\pi_f(p)] < E[\pi_f(r_f)]$ if $p \in (r_f, r_f+\epsilon_1)$. This results in a "hole" $(r_f, r_f+\epsilon_1)$ for the fringe retailer when $p < r_d$. It contradicts lemma 2. Thus, $\overline{p}_f \leq r_f$ under a mixed strategy equilibrium.

This proves lemma 3. Q.E.D

Proof of Proposition 1

First, let's establish some properties regarding to the mass point of equilibrium price distribution:

(1) There must exist a mass point. (e.g., Narasimhan 1988; Koçaş and Kiyak 2006).

(2) All retailers cannot have a mass point at the same price. Otherwise, one retailer is better off from moving probability mass to a slightly lower price than the mass point. This gives it a discontinuous increase in demand without sharing shoppers at the mass point.

(3) The mass point can only exist at the reservation prices. We prove this by contradiction. Let's consider the case $r_f < r_d$, note that the same proof goes through the opposite case.

   a. First, no retailer can have a mass point at $p < r_f < r_d$. If one retailer has a mass point at $p < r_f < r_d$, the other one has a higher profit at $p + \epsilon$, where $\epsilon \to 0$ and $\epsilon > 0$, than at p, because it has a higher margin and the same demand. This contradicts the definition of mixed strategy equilibrium.

   b. Second, no mass point can exist for $p \in (r_f, r_d)$. Since $\overline{p}_f \leq r_f$, all non-shoppers purchase from the first retailer that they visit. Consequently, the prominent retailer has no incremental demand from pricing at $p \in (r_f, r_d)$ than $p = r_d$. As a result, it has a strictly lower profit at $p \in (r_f, r_d)$ than $p = r_d$. This implies that the prominent retailer puts zero probability at $p \in (r_f, r_d)$. Hence force, there can be no mass point at $p \in (r_f, r_d)$.

Next, let's consider the lower boundary of price support. Under the full awareness, we must have $\underline{p}_d \geq \underline{p}_f$. The intuition is as follows. In this case, two symmetric fringes and one prominent retailer compete for shoppers. When one fringe retailer prices at $\underline{p}_d$, it does not capture shoppers with probability one, because the other fringe's price might be lower than $\underline{p}_d$. As a result, the fringe has an incentive to price below $\underline{p}_d$. However, by pricing at $\underline{p}_f$, the prominent retailer undercuts the prices of both fringes. Therefore, the prominent retailer never prices below $\underline{p}_f$. The above logic implies that $\underline{p}_d \geq \underline{p}_f$.

Now let's consider the upper boundary. Given that the search cost equals search benefit: $c = \int_{\underline{p}_f}^{r_d} F_f(p)dp = \max\left\{\int_{\underline{p}_d}^{r_f} F_d(p)dp, \int_{\underline{p}_f}^{r_f} F_f(p)dp\right\}$, there can be two possible cases: (1) if $\int_{\underline{p}_d}^{r_f} F_d(p)dp \leq \int_{\underline{p}_f}^{r_f} F_f(p)dp = \int_{\underline{p}_f}^{r_d} F_f(p)dp$, then $r_d = r_f = r$; (2) if $\int_{\underline{p}_f}^{r_d} F_f(p)dp = \int_{\underline{p}_p}^{r_f} F_d(p)dp > \int_{\underline{p}_f}^{r_f} F_f(p)dp$, then $r_d > r_f$. We consider both and find that equilibrium exits only for the first case.

Case 1: $\mathbf{r_d = r_f = r}$

The proof proceeds as follows. First, we identify the mass point and use it to find the equilibrium profit of one retailer. Second, we locate the lower boundary of the price support and find the equilibrium price distribution as a function of the reservation price. Lastly, we solve the reservation price and very that it is rational for the consumer start search at her default retailer.

Step 1. In this case, a mass point exists at r of either $F_d(p)$ or $F_f(p)$ but not at both of them. This implies that at least one of the fringe's competitors (either the prominent or the other fringe retailer) does not have a mass point at r. Thus, the fringe retailer captures shoppers with zero probability at r. This gives us the fringe retailer's equilibrium profit as $E\pi_f = \pi_f(r) = \frac{(1-\mu)(1-\alpha_d)}{2} r$. In addition, the prominent retailer's competitors might have a mass point at r. Consequently, the prominent retailer captures shoppers either with positive or zero probability at r, this gives $E\pi_d = \pi_d(r) \geq (1-\mu)\alpha_d r$.

Step 2. Because $E\pi_f = \pi_f\left(\underline{p}_f\right)$, we can have $\underline{p}_f = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+2\mu} r$. Given that it is assured of getting the entire shopper segment, the lowest price that a prominent retailer is willing to charge is no higher than $\frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r$, because $E\pi_d \geq (1-\mu)\alpha_d r$. This gives $\underline{p}_d > \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r$. In addition, we have $\underline{p}_d > \underline{p}_f$ since $\underline{p}_f < \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r$. It implies that $F_d(p) = 0$ for $p \in [\underline{p}_f, \underline{p}_d]$. This gives the fringe retailer's profit for $p \in [\underline{p}_f, \underline{p}_d]$ as

$$E\pi_f = \left\{\frac{(1-\mu)(1-\alpha_d)}{2} + \mu[1 - F_f(p)]\right\} p.$$

Substitute $E\pi_f = \frac{(1-\mu)(1-\alpha_d)}{2} r$ into it, we have $1 - F_f(p) = \frac{(1-\mu)(1-\alpha_d)}{2\mu}\left(\frac{r}{p} - 1\right)$ for $p \in [\underline{p}_f, \underline{p}_d]$.

Next, let's solve $\underline{p}_d$. Since $\pi_d\left(\underline{p}_d\right) = E\pi_d \geq (1-\mu)\alpha_d r$, we have:

$$\left\{(1-\mu)\alpha_d + \mu\left[1 - F_f\left(\underline{p}_d\right)\right]^2\right\}\underline{p}_d \geq (1-\mu)\alpha_d r.$$

Substitute $1 - F_f\left(\underline{p}_d\right) = \frac{(1-\mu)(1-\alpha_d)}{2\mu}\left(\frac{r}{\underline{p}_d} - 1\right)$ into it, we have

$$\frac{1}{\mu}\left[\frac{(1-\mu)(1-\alpha_d)}{2}\left(\frac{r}{\underline{p}_d} - 1\right)\right]^2 \geq (1-\mu)\alpha_d\left(\frac{r}{\underline{p}_d} - 1\right).$$

From it, we have either $\underline{p}_d = r$ or $\underline{p}_d \leq \frac{(1-\mu)(1-\alpha_d)^2}{4\alpha_d\mu+(1-\mu)(1-\alpha_d)^2} r$. However, $\frac{(1-\mu)\alpha_d}{\mu+(1-\mu)\alpha_d} r > \frac{(1-\mu)(1-\alpha_d)^2}{4\alpha_d\mu+(1-\mu)(1-\alpha_d)^2} r \geq \underline{p}_d$ for $\alpha_d > \frac{1}{3}$. This contradicts to $\underline{p}_d \geq \frac{(1-\mu)\alpha_d}{\mu+(1-\mu)\alpha_d} r$. Therefore, $\underline{p}_d = r$.

Step 3. We have $r = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}$ after substituting $F_f(p)$ and $\underline{p}_f$ into $c = \int_{\underline{p}_f}^{r} F_f(p)dp$. We can verify that each consumer's expected utility from starting at her default is ***weakly higher*** than non-default retailers. For consumers who take the prominent retailer as the default, we have $EU_d = v - Ep_d - 0 = v - r \geq EU_f = v - Ep_f - c = v - r$. For those who take the fringe as the default, we have $EU_f = v - Ep_f - 0 = v - r + c \geq EU_d = v - Ep_d - c \geq v - r - c$.

Case 2: $r_d > r_f$

In this case, $\int_{\underline{p}_d}^{r_f} F_d(p)dp > \int_{\underline{p}_f}^{r_f} F_f(p)dp$. Given the mass point property of asymmetric equilibrium, we have three possible subcases: (1) only $F_d(p)$ has a mass point at r_f; (2) $F_f(p)$ and $F_d(p)$ have a mass point at r_f and r_d, respectively; (3) only $F_f(p)$ has a mass point at r_f. We will show that no equilibrium holds for any of the above cases.

Subcase 2.1. We rule out this case by showing that it contradicts the definition of mixed strategy equilibrium. If a mass point exists at r_f of $F_d(p)$, then $F_f(p)$ cannot have a mass point at r_f. Consequently, the prominent retailer captures shoppers with zero probability at both r_f and r_p. Since $r_d < r_p$ we have $E\pi_d(r_f) = (1-\mu)\alpha_d r_f < E\pi_d(r_d) = (1-\mu)\alpha_d r_d$. This contradicts the definition of mixed strategy equilibrium. Hence, no equilibrium holds in this case.

Subcase 2.2. We show that there is unilateral deviation in this case. If there exists a mass point at r_d of $F_d(p)$, then $F_d(r_f) < 1$ because $r_f < r_d$. If both fringes have mass points at r_f, then one of them is better off from shifting mass to a price slightly below r_f. This yields a discontinuous demand increase by not sharing shoppers with the competitors at r_f. Therefore, no equilibrium holds in this case.

Subcase 2.3. Following the same process in case 1, we can try to solve the equilibrium price distribution and reservation price. However, similar to case 1, we find that the prominent retailer plays a pure strategy at r_f in this case. However, this would lead to unilateral deviation. Given both fringes have mass point at r_f and the prominent retailer plays a pure strategy at r_f, one fringe becomes strictly better off from shifting mass to a price slightly below r_f. This yields a discontinuous demand increase by not sharing shoppers with the competitors at r_f. Hence, no equilibrium holds in this case. The remaining proof is the same as that of case 1 except that we replace r with r_f, we do not show the repeated proof due to the limited space. Q.E.D.

Proof of Proposition 2

In this section, we show that all retailer's average prices, Ep_d and Ep_f, decreases with α_d for any $\alpha_d > \frac{1}{3}$.

First, we have $Ep_d = r = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]} = \frac{c}{1-A(\alpha_d)}$.

$\frac{\partial A(\alpha_d)}{\partial \alpha_d} = \frac{(1-\mu)}{(1-\mu)(1-\alpha_d)+2\mu} - \frac{(1-\mu)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right] < 0$, because $\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right] > \frac{2\mu}{(1-\mu)(1-\alpha_d)+2\mu}$. This proves that $\frac{\partial A(\alpha_d)}{\partial \alpha_d}$ and thus $\frac{\partial Ep_d}{\partial \alpha_d} < 0$.

Second, we have $Ep_f = \int_{\underline{p}}^{r} p\frac{dF_f(p)}{dp}dp = pF_f(p)|_{\underline{p}}^{r} - \int_{\underline{p}}^{r} F_f(p)dp = r - c$. As such, $\frac{\partial Ep_f}{\partial \alpha_d} = \frac{\partial r}{\partial \alpha_d} < 0$. Q.E.D.

Proof of Proposition 3

In this section, we show that consumer welfare can increase with search traffic concentration α_d. We focus on how the consumer's consumption utility changes with α_d, because consumer's search pattern does not change with respect to α_d. Consider a consumer who purchases at retailer i ($i = d, f$), her expected consumption utility is $EU_i = v - Ep_i$. As we have shown previously, for any i, $\frac{\partial Ep_i}{\partial \alpha_d} < 0$. As such, we have $\frac{\partial EU_i}{\partial \alpha_d} = -\frac{\partial Ep_i}{\partial \alpha_d} > 0$. This implies that consumer welfare increases with search traffic concentration. Q.E.D.

Proof of Proposition 4

We will show that that $E\pi_d$ decreases with α_d when μ is sufficiently low and $\alpha_d > \frac{1}{2}$ as follows.

After substituting $F_f(p)$ and $\underline{p}_f$ into $E\pi_d$, we have $E\pi_d = \frac{c}{\frac{1}{(1-\mu)\alpha_d} - \frac{(1-\alpha_d)}{2\alpha_d\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}$.

Let $E\pi_d = \frac{c}{n(\alpha_d)}$, where $n(\alpha_d) = \frac{1}{(1-\mu)\alpha_d} - \frac{(1-\alpha_d)}{2\alpha_d\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]$. We can show that $E\pi_d$ decreases with α_d when μ is sufficiently low and $\alpha_d > \frac{1}{2}$ by proving that $n(\alpha_d)'$ decreases with α_d when μ is sufficiently low and $\alpha_d > \frac{1}{2}$ as follows.

We have $n(\alpha_d)' = -\frac{1+\mu}{\alpha_d^2(1-\mu)[(1-\mu)(1-\alpha_d)+2\mu]} + \frac{\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}{2\mu\alpha_d^2}$. Notice that we do not have a closed-form solution for $n(\alpha_d)' > 0$. Instead, we can show this numerically. Figure A1 depicts that $n(\alpha_d)' > 0$ in the shaded area. From it, we know that $n(\alpha_d)' > 0$ when μ is sufficiently low relative to α_d and $\alpha_d > \frac{1}{2}$. Consequently, $\frac{\partial E\pi_d}{\partial \alpha_d} = -\frac{c}{n^2(\alpha_d)}n(\alpha_d)' < 0$ when μ is sufficiently low relative to α_d and $\alpha_d > \frac{1}{2}$.

<Insert Figure A1 Here>

Q.E.D

Proof of Corollary 1

We want to show that that $E\pi_d$ increases with α_d when μ is sufficiently high or $\alpha_d > \frac{1}{2}$. This can be directly shown by Figure A1 in Proof of Proposition 4. As $n(\alpha_d)' \leq 0$ in the light area in

Figure A1, we must have $\frac{\partial E\pi_d}{\partial \alpha_d} = -\frac{c}{n^2(\alpha_d)} n(\alpha_d)' \geq 0$ when μ is sufficiently high relative to α_d or $\alpha_d \leq \frac{1}{2}$. Q.E.D.

Proof of Proposition 5

First, consider the lower boundary of price support. In this case, each shopper searches one prominent and one fringe retailer. Thus, the prominent retailer only needs to undercut one fringe retailer's price to acquire shoppers, and vice versa. Consequently, no retailer is willing to charge a price that is lower than the competitor's lower boundary. Therefore, two retailers share the common lower boundary, $\underline{p}_d = \underline{p}_f = \underline{p}$. Note that this is similar to the duopoly competition in Narasimhan (1988). By pricing at the common boundary $\underline{p}$, a retailer captures shoppers with probability one.

Regarding to the upper boundary, there can be two possible cases: (1) $r_d \leq r_f$ and (2) $r_d > r_f$. We consider both and find that equilibrium holds for the first one when $\alpha_d = \alpha^* \geq \frac{1}{2}$ and for the latter case when $\alpha_d < \alpha^*$.

Case 1:$\mathbf{r_d \leq r_f}$

If $r_d \leq r_f$, we have $\int_{\underline{p}}^{r_d} F_d(p)dp \leq \int_{\underline{p}}^{r_d} F_f(p)dp$, because $\int_{\underline{p}}^{r_d} F_d(p)dp \leq \int_{\underline{p}}^{r_f} F_d(p)dp = \int_{\underline{p}}^{r_d} F_f(p)dp = c$. The rest of the proof in this case proceeds as the follows.

First, we find there can be two possible subcases, which depends on whether a mass point exists at r_f of $F_f(p)$ or not.

Second, we derive the equilibrium outcome in both subcases and find that equilibria exist only when $\alpha_d \geq \frac{1}{2}$. The derivation follows four steps. Step 1, identify the mass point. Step 2, derive the equilibrium price distribution given reservation price. Step 3, make sure that $\int_{\underline{p}}^{r_d} F_d(p)dp \le \int_{\underline{p}}^{r_d} F_f(p)dp$. Step 4, solve the reservation price from optimal stopping rule.

Finally, we select the equilibrium that yields higher profits for both retailers. We find the selected equilibrium has no mass point at r_f for $F_f(p)$. The intuition is as follows. No mass point at r_f implies a lower competition level. Thus, both retailers have higher profits in this scenario.

Subcase 1.1: $\boldsymbol{F_f(p)}$ has no mass point at $\boldsymbol{r_f}$

Step 1. If no mass point exists at r_f of $F_f(p)$, then either $F_d(p)$ or $F_f(p)$ has a mass point at r_d. We show that $F_f(p)$ does NOT have a mass point at r_d by contradiction. If $F_f(p)$ has a mass point at r_d, then $F_d(p)$ does not have a mass point at r_d. This implies that the fringe retailer captures shoppers with zero probability at both r_d and r_f since $r_d \le r_f$. This gives $\pi_f(r_d) = \frac{(1-\mu)(1-\alpha_d)}{2} r_p \le \pi_f(r_f) = \frac{(1-\mu)(1-\alpha_d)}{2} r_f$.[11] However, this contradicts the definition of mixed strategy equilibrium. As a result, $F_f(p)$ does not have a mass point at r_d.

Step 2. Since $F_f(p)$ has no mass point, the prominent retailer gets shoppers with zero probability at r_d. This gives $E\pi_d = \pi_d(r_d) = (1-\mu)\alpha_d r_d$. Substitute it into (4.1), we have $F_f(p) = 1 - \frac{(1-\mu)\alpha_d}{\mu}(\frac{r_d}{p} - 1)$ for $p \in [\underline{p}, r_d]$. Given $E\pi_d = \pi_d(\underline{p}) = [(1-\mu)\alpha_d + \mu]\underline{p}$, we have $\underline{p} = \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_d$. This gives $E\pi_f = \pi_f\left(\underline{p}\right) = \left[\frac{(1-\mu)(1-\alpha_d)}{2} + \frac{\mu}{2}\right]\frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_d$. Substitute it into (4.2), we have $F_d(p) = \frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}]$ for $p \in [\underline{p}, r_d]$.

Step3. We show that this equilibrium only holds when $\alpha_d \ge \frac{1}{2}$ as follows. When $\alpha_d < \frac{1}{2}$, we find that $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_d} F_f(p)dp$, because $F_d(p) - F_f(p) = \frac{(1-2\alpha_d)(1-\mu)}{\mu}\left[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}\right] > 0$ for $p \in [\underline{p}, r_d]$, which contradicts to $\int_{\underline{p}}^{r_d} F_d(p)dp \le \int_{\underline{p}}^{r_d} F_f(p)dp$.

Step 4. We have $r_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]}$ and $r_f = r_d + \frac{(1-\mu)(2\alpha_d-1)}{(1-\mu)\alpha_d+\mu}c$ after substituting $F_f(p)$, $F_d(p)$, and $\underline{p}$ into $c = \int_{\underline{p}}^{r_d} F_f(p)dp = \int_{\underline{p}}^{r_f} F_d(p)dp$. Given r_d and r_f, we have $E\pi_d = \frac{c}{-\frac{1}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]+\frac{1}{(1-\mu)\alpha_d}}$ and $E\pi_f = \frac{(1-\mu)(1-\alpha_d)+\mu}{-\frac{(1-\mu)\alpha_d+\mu}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]+\frac{(1-\mu)\alpha_d+\mu}{(1-\mu)\alpha_d}}\frac{c}{2}$.

Subcase 1.2: $\boldsymbol{F_f(p)}$ *has a mass point at* $\boldsymbol{r_f}$

Step 1. If $F_f(p)$ has a mass point at r_f, $F_d(p)$ must have a mass point at r_d. If no mass point exists at r_d for $F_d(p)$, the fringe retailer captures shoppers with zero probability at both r_d and r_f. Since $r_d \leq r_f$, $\pi_d(r_d) \leq \pi_f(r_f)$. This violates the definition of mixed strategy equilibrium. Note that we define the break-even condition to violate the mixed strategy equilibrium definition. It would not change the result qualitatively. Therefore, a mass point must exist at r_d. In this case, given that the prominent retailer's price is no higher than r_d, the fringe retailer puts no weight on $p \in (r_d, r_f)$, under which it has a strictly lower profit than $p = r_f$ due to the lower margin yet the same demand.

Step 2. Since the fringe retailer gets shopper with zero probability at r_f, We have $E\pi_f = \pi_f(r_f) = \frac{(1-\mu)(1-\alpha_d)}{2}r_f$. Substitute it into (4.2), We have $F_d(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}(\frac{r_f}{p} - 1)$ for $p \in [\underline{p}, r_d]$. Given $\pi_f(\underline{p}) = E\pi_f$, this yields $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}r_f$. Substitute $\underline{p}$ into $\pi_d\left(\underline{p}\right)$, we have $E\pi_d = \pi_d\left(\underline{p}\right) = [(1-\mu)\alpha_d + \mu]\frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}r_f$. This gives $F_f(p) = \frac{(1-\mu)\alpha_d+\mu}{\mu}[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}]$ for $p \in [\underline{p}, r_d]$.

Step 3. We can show that this equilibrium holds only when $\alpha_d \geq \frac{1}{2}$ as follows. When $\alpha_p < \frac{1}{2}$, $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_d} F_f(p)dp$, because $F_d(p) - F_f(p) = \frac{(1-2\alpha_d)(1-\mu)}{\mu}\left[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}\right] > 0$ for $p \in [\underline{p}, r_d]$. This contradicts to $\int_{\underline{p}}^{r_d} F_d(p)dp \leq \int_{\underline{p}}^{r_d} F_f(p)dp$.

Step 4. We cannot have closed form solution for the reservation price and thus the profit in this case. Instead, let's derive the upper bound of retailers' profits in this case. Substitute $F_i(p)$ and $\underline{p}$ into $c = \int_{\underline{p}}^{r_f} F_d(p)\,dp$ and $c = \int_{\underline{p}}^{r_d} F_f(p)\,dp$, we have

$$(1-\mu)(1-\alpha_d)r_f \ln\left[\left(1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right)\frac{r_f}{r_d}\right] = \mu c - (1-\mu)(1-\alpha_d)r_d - [\mu - (1-\mu)(1-\alpha_d)]r_f.$$

$$(1-\mu)(1-\alpha_d)r_f \ln\left[\left(1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right)\frac{r_f}{r_d}\right] = \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}\mu c - [(1-\mu)(1-\alpha_d)+\mu]r_d + (1-\mu)(1-\alpha_d)r_f.$$

Combining the two equations, we have

$$r_f - r_d = \frac{(1-\mu)(2\alpha_d-1)}{(1-\mu)\alpha_d+\mu}c.$$

Since $F_f(p)$ has a mass point at r_f, we have $F_f(r_f) < 1$. This gives $F_f(r_d) < 1$ given $r_d \leq r_f$. Consequently, $E\pi_d = \pi_d(r_f) = \{(1-\mu)\alpha_d + \mu[1 - F_f(r_d)]\}p > (1-\mu)\alpha_d r_d$. It yields $\pi_d\left(\underline{p}\right) = [(1-\mu)\alpha_d + \mu]\frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}r_f = E\pi_d > (1-\mu)\alpha_d r_d$. Substitute $r_d = r_f - \frac{(1-\mu)(2\alpha_d-1)}{(1-\mu)\alpha_d+\mu}c$ into it, we have $r_f < \frac{(1-\mu)\alpha_d[(1-\mu)(1-\alpha_d)+\mu]}{\mu[(1-\mu)\alpha_d+\mu]}c$. Hence, we have $E\pi_d < \frac{(1-\mu)^2(1-\alpha_d)\alpha_d}{\mu}c$ and $E\pi_f < \frac{(1-\mu)^2(1-\alpha_d)\alpha_d[(1-\mu)(1-\alpha_d)+\mu]}{2\mu[(1-\mu)\alpha_d+\mu]}c$.

Profit Comparison and Equilibrium Selection

We show that both retailers have higher profits under subcase 1.1 than 1.2 as follows. Suppose retailer i's profit in 1.1 as $E\pi_i'$. We have $E\pi_d' > \frac{(1-\mu)\alpha_d[(1-\mu)\alpha_d+\mu]}{\mu}c$, because $\ln(1+x) > \frac{x}{1+x}$ where $x = \frac{\mu}{(1-\mu)\alpha_p}$. When $\alpha_d \geq \frac{1}{2}$, $\frac{E\pi_d'}{E\pi_d} > \frac{(1-\mu)\alpha_d+\mu}{(1-\mu)(1-\alpha_d)} > 1$, because $E\pi_d <$

$\frac{(1-\mu)^2(1-\alpha_d)\alpha_d}{\mu}c$. Following the same proof, we can show that $\frac{E\pi_f'}{E\pi_f} > 1$. Therefore, we choose the equilibrium in 1.1, under which both retailers have higher profits.

As a summary, when $\alpha_d \geq \frac{1}{2}$, $F_d(p) = \frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}]$ and $F_f(p) = 1 - \frac{(1-\mu)\alpha_d}{\mu}(\frac{r_d}{p} - 1)$ $p \in [\underline{p}, r_d]$, where $r_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln[1+\frac{\mu}{(1-\mu)\alpha_d}]}$ and $\underline{p} = \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}r_d$. In this case, $Ep_d = r_d - \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}c$ and $Ep_f = r_d - c$.

Lastly, we can verify that each consumer's expected utility from starting at her default is ***weakly higher*** than non-default retailers as follows. For consumers who take the prominent retailer as the default, we have $EU_d = v - Ep_d - 0 = v - r_d + \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}c \geq EU_f = v - Ep_f - c = v - r_d$. For those who take the fringe as the default, we have $EU_f = v - Ep_f - 0 = v - r_d + c \geq EU_d = v - Ep_d - c = v - r_d - \frac{(1-\mu)(2\alpha_d-1)}{(1-\mu)\alpha_d+\mu}c$.

Case 2: $r_d > r_f$

If $r_d > r_f$, we have $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_d} F_f(p)dp$, because $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_f} F_d(p)dp = \int_{\underline{p}}^{r_d} F_f(p)dp$. Similar to the previous case, there are two subcases. Following the same procedure in case 1, we derive the equilibrium in both subcases. We find that equilibria exist only when $\alpha_d < \frac{1}{2}$ in order to satisfy $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_d} F_f(p)dp$. Then we choose the equilibrium under which both retailers have higher profits. Due to the limited space, we leave the detailed proof in web appendix section b. Here we report the equilibrium outcome as follows. When $\alpha_d < \frac{1}{2}$, $F_d(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}(\frac{r_f}{p} - 1)$ and $F_f(p) = \frac{(1-\mu)\alpha_d+\mu}{\mu}[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}]$ for $p \in [\underline{p}, r_f]$, where $r_f = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}$ and $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}r_f$. Q.E.D.

Proof of Proposition 6

Impact of α_d on Ep_i

In this section, we show that $\frac{\partial Ep_i}{\partial \alpha_d} > 0$ when $\alpha_d \geq \alpha^* = \frac{1}{2}$ and $\frac{\partial Ep_i}{\partial \alpha_d} < 0$ when $\alpha_d < \alpha^*$. First, we have $Ep_i = \int_{\underline{p}}^{r_i} p \frac{dF_i(p)}{dp} dp = pF_i(p)|_{\underline{p}}^{r_i} - \int_{\underline{p}}^{r_i} F_i(p)dp = r_i - \int_{\underline{p}}^{r_i} F_i(p)dp$.

When $\alpha_d \geq \alpha^*$

When $\alpha_d > \alpha^*$, we have $r_d = \frac{c}{g(\alpha_d)}$, where $g(\alpha_d) = 1 - \frac{(1-\mu)\alpha_d}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right]$. $g'(\alpha_d) = -\frac{\mu}{1-\mu}\left\{\ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right] - \frac{\mu}{(1-\mu)\alpha_d+\mu}\right\} < 0$, because $\ln(1+x) > \frac{x}{1+x}$ where $x = \frac{\mu}{(1-\mu)\alpha_d}$. This gives $\frac{\partial r_d}{\partial \alpha_d} = -\frac{g'(\alpha_d)}{g(\alpha_d)^2} c > 0$.

Also, we have $Ep_d = r_d - \int_{\underline{p}}^{r_d} F_d(p)dp = r_d - \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu} r_d \left\{1 - \frac{(1-\mu)\alpha_d}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right]\right\}$. This gives $Ep_d = r_d - \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu} c$ given $c = r_d\left\{1 - \frac{(1-\mu)\alpha_d}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right]\right\}$.

Hence, $\frac{\partial Ep_d}{\partial \alpha_d} = \frac{\partial r_d}{\partial \alpha_d} + \frac{(1-\mu)(1+\mu)}{[(1-\mu)\alpha_d+\mu]^2} c > 0$.

Moreover, $Ep_f = r_f - \int_{\underline{p}}^{r_f} F_f(p)dp = r_f - \{\int_b^{r_d} F_f(p)dp + \int_{r_d}^{r_f} 1dp\} = r_d - c$.

Consequently, $\frac{Ep_f}{\partial \alpha_d} = \frac{\partial r_d}{\partial \alpha_d} > 0$.

When $\alpha_d < \alpha^*$

In this case, we have $r_f = \frac{c}{h(\alpha_d)}$, where $h(\alpha_d) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right]$. $h'(\alpha_d) = \frac{1-\mu}{\mu}\left\{\ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right] - \frac{\mu}{(1-\mu)(1-\alpha_d)+\mu}\right\} > 0$, because $\ln(1+x) > \frac{x}{1+x}$ where $x = \frac{\mu}{(1-\mu)(1-\alpha_d)}$.

Therefore, $\frac{\partial r_f}{\partial \alpha_d} = -\frac{h'(\alpha_d)}{h^2(\alpha_d)} c < 0$.

Furthermore, we have $Ep_d = r_d - \int_{\underline{p}}^{r_d} F_d(p)dp = r_d - \{\int_{\underline{p}}^{r_f} F_d(p)dp + \int_{r_f}^{r_d} 1dp\} = r_f - c$. It yields $\frac{\partial Ep_d}{\partial \alpha_d} = \frac{\partial r_f}{\partial \alpha_d} < 0$.

Lastly, $Ep_f = r_f - \int_{\underline{p}}^{r_f} F_f(p)dp = r_f - \frac{(1-\mu)\alpha_d+\mu}{(1-\mu)(1-\alpha_d)+\mu} r_f \left\{1 - \frac{(1-\mu)(1-\alpha_d)}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right]\right\}$. Given $c = r_f \left\{1 - \frac{(1-\mu)(1-\alpha_d)}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right]\right\}$, we have $Ep_f = r_f - \frac{(1-\mu)\alpha_d+\mu}{(1-\mu)(1-\alpha_d)+\mu} c$ This gives $\frac{\partial Ep_f}{\partial \alpha_d} = \frac{\partial r_f}{\partial \alpha_d} - \frac{(1-\mu)(1+\mu)}{[(1-\mu)(1-\alpha_d)+\mu]^2} c < 0$.

Impact of α_d on consumer welfare EU_i

Consider a consumer who purchases at retailer i, her expected consumption utility is $EU_i = v - Ep_i$. As shown previously, $\frac{\partial Ep_i}{\partial \alpha_d} > 0$ when $\alpha_d \geq \alpha^* = \frac{1}{2}$ and $\frac{\partial Ep_i}{\partial \alpha_d} < 0$ when $\alpha_d < \alpha^*$. This gives us $\frac{\partial EU_i}{\partial \alpha_d} < 0$ when $\alpha_d \geq \alpha^* = \frac{1}{2}$ and $\frac{\partial EU_i}{\partial \alpha_d} > 0$ when $\alpha_d < \alpha^*$.

Impact of α_d on $E\pi_d$

In this section, we show that $\frac{\partial E\pi_d}{\partial \alpha_d} < 0$ when $\alpha_d < \frac{1}{2}$ and $\mu > \sqrt{\frac{1}{2} + \frac{\alpha_d^2}{16(1-\alpha_d)^2}} - \frac{\alpha_d}{4(1-\alpha_d)}$. When $\alpha_d < \frac{1}{2}$, $\frac{\partial E\pi_d}{\partial \alpha_d} = \frac{c}{\left\{-\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]\frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}+1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right\}^2} l(\mu, \alpha_d)$, where $l(\mu, \alpha_d) = \frac{1+\mu}{1-\alpha_d} - \frac{\mu[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)^2} - \frac{(1-\mu)(1+\mu)}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right] < \frac{1+\mu}{1-\alpha_d} - \frac{\mu[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)^2} = \frac{2(\alpha_d-1)\mu^2-\alpha_d\mu+(1-\alpha_d)}{(1-\mu)(1-\alpha_d)^2}$, because $\ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right] > 0$. This gives us a sufficient condition for $l(\mu, \alpha_d) < 0$: $2(\alpha_d - 1)\mu^2 - \alpha_d\mu + (1 - \alpha_d) < 0$. From it, we have $\mu > \mu_1(\alpha_d) = \sqrt{\frac{1}{2} + \frac{\alpha_d^2}{16(1-\alpha_d)^2}} - \frac{\alpha_d}{4(1-\alpha_d)}$.

Consequently, if $\mu > \mu_1$, $l(\mu, \alpha_d) < 0$ and thus $\frac{\partial E\pi_d}{\partial \alpha_d} < 0$. Q.E.D.