Doing Battle with Short Sellers: The Role of Blockholders in Bear Raids

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Abstract. If short sellers can destroy firm value by manipulating prices down in a "bear raid," an informed blockholder has a powerful natural incentive to protect the value of his stake by trading against them. However, he also has an incentive to use his information to generate trading profits. We show that these conflicting objectives create a multiplier effect, whereby the buying quantity needed to defeat the shorts becomes a large multiple of the expected amount of short selling. This increases trading profits when the blockholder buys at favorable prices, but also increases losses when he must buy at unfavorable prices. Thus, his existing stake needs to be large enough to absorb these losses. Importantly, though, the multiplier shrinks as the potential for value destruction increases, meaning a smaller stake is sufficient precisely when a successful bear raid would be most harmful. These results add a new dimension to the existing debate on when/whether intervention against short sellers is warranted.

Keywords: speculation, short selling, regulation, manipulation, bear raids

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1. Introduction

Recent events have added urgency to the ongoing debate over the costs and benefits of short selling activity. On one side of this debate are those who believe short sellers manipulate prices for personal gain, creating lasting problems for the targeted firms. On the other side are those who believe short sellers bring important information to the market, preventing stocks from being over-valued and making the market more liquid. In this paper we add a new element to this debate. We argue that any discussion about the potential damaging role of short sellers should also consider the actions of another class of important participants in the market, namely informed blockholders who maintain long positions in the firms' stock. If there is reason to believe that short sellers may cause lasting negative effects by manipulating prices down, such blockholders have powerful natural incentives to prevent such manipulation. They can do so by buying enough shares to keep prices high, and may be willing to do so even if that necessitates buying shares at unfavorable prices and incurring trading losses. Thus, private markets may be able to handle value-destroying attempts by speculators without outside help, and the beneficial effects of short selling may dominate.

The idea that short sellers' price manipulations can create lasting damage is clearly expressed by the SEC in its defense of the September 2008 short sale ban. A press release dated September 19th states "it appears that unbridled short selling is contributing to the recent, sudden price declines in the securities of financial institutions unrelated to true market valuation." The release goes on to say that such price declines are capable of causing a "crisis of confidence ... because they (institutions) depend on the confidence of their trading counterparties in the conduct of their core business." A similar idea has been captured by the academic literature on "feedback effects," in which large stock price movements induce permanent changes in fundamental value through their impact on decisions affecting the firm.[1] In the context of the recent economic crisis, this type of reverse causality is likely, for example, when decision makers like creditors or other counterparties depend on the firm's stock price to infer important information about its prospects.[2] In such situations, these decision makers may be less willing to establish or continue valuable relationships with the firm following a significant price drop. Thus the damage may be caused not so much by the change in stock price, but through its feedback effect on the real decisions of the firm's counterparties, since that not only amplifies the price change but makes it permanent.

We incorporate both the presence of an informed long-term blockholder and the presence of a feedback effect in a model of a potential bear raid by a short seller. In particular, we study a firm whose value is affected by a decision maker's choice of whether to accept or reject a counterparty relationship with it. A risk neutral long-term blockholder/investor holds a long position in the firm's stock and possesses private information about the firm's prospects which is valuable to the decision maker, but can be credibly conveyed only through trading in the stock market.[3] Market prices are set based on net order flows by a risk neutral and wealth unconstrained market maker as in Glosten and Milgrom (1985) and Kyle (1985).

[1]Several recent papers in this literature specifically focus on how feedback effects may give rise to manipulation, including Khanna and Sonti (2004), Attari, Banerjee, and Noe (2006), and Goldstein and Guembel (2008), the last of which focuses on manipulative short selling. See pages 4-5 for a full discussion.

[2]See, e.g., Durnev, Morck, and Yeung (2005), Luo (2005), Sunder (2005), Bakke and Whited (2008), Chen, Goldstein, and Jiang (2007), and Edmans, Goldstein, and Jiang (2008) for evidence of managers, creditors, and other counterparties making decisions in part based on stock prices.

[3]A question arises as to whether direct communication with the decision maker could solve the underlying problems. However, in our model it turns out that the blockholder does not want to fully reveal his information either to the market or to the decision maker because of both his incentive to make trading profits and his incentive to get the right decision made. Furthermore, since the decision maker resides outside the firm, and the single decision maker we model may actually represent numerous such agents across different counterparties, a direct communication mechanism may be infeasible in practice.

We first show that full efficiency (i.e., the acceptance of all positive net present value relationships by the decision maker) is not guaranteed even in the absence of a speculator who could attempt a bear raid. The reason is that the investor has two potentially competing objectives in his trading strategy. First, he wants to ensure that the decision maker makes an efficient decision so that the value of his existing stake in the firm is maximized. Second, he wants to use his information to maximize his trading profits (or minimize his trading losses). Given a base level of noise trade in the market, the incentive to maximize trading profits when his information indicates a highly profitable relationship sets an endogenous lower bound on the trading quantity that is required to convince the decision maker to accept the relationship. However, this creates a problem for the investor if his information indicates that the relationship, while still valuable, is not as profitable, because in this case he may be forced to buy the required quantity at prices he knows are too high given his information. He will be willing to do so only if his initial stake is large enough that the gain to its value from ensuring the acceptance of the relationship justifies incurring the necessary trading losses.

Next consider how an uninformed speculator can potentially profit in this framework. She observes that noise in the stock market generates inefficiency, causing some profitable relationships to be lost. We show that she can profit by trading in a way that exacerbates this problem, leading to a "multiplier effect" whereby the trading quantity required for the investor to convince the decision maker to accept the relationship becomes a large multiple of the amount of potential short selling. In essence, if the speculator can arrive with a hidden long or short initial position, and then (optimally) trade against the informed trader in the direction of her position, she is able to bring the investor's twin objectives into greater conflict. Thus, when she is short she effectively "raids" relationships with moderate expected profitability in an attempt to cause their rejection and destroy value.

This implies that the speculator's actions create an "efficiency gap" in that significantly larger shareholdings by informed long-term investors are required to ensure the efficient outcome. If the actual holdings fall within this gap, the speculator's actions can reduce firm value (by causing some inefficient rejections), potentially generating profits for her. Consistent with real life trading, we assume the ability to short sell is limited, so the efficiency gap we derive is measured *relative* to these limits.[4] The reason that even a relatively restricted short seller can sometimes profitably manipulate in our setting is because of the endogenous constraint the investor's twin objectives impose on his willingness to trade against her.

It is important to note, however, that only moderately profitable relationships can be successfully raided in our setting. Furthermore, we show that the size of the stake needed to ensure efficiency shrinks as the potential loss in value from a bear raid increases. This occurs both because the blockholder's incentive to prevent bear raids increases, *and*, surprisingly, because the expected trading losses required to implement the strategy *decrease*. This shrinks the multiplier, and thus the needed size of the block, making it more likely that bear raids will be prevented precisely when they would be most harmful.[5]

These findings suggest that in the presence of a large blockholder, the role of outside intervention is limited. However, significant short selling abuses arguably exist in practice, which if true implies that blockholders are sometimes choosing not to hold sufficiently large stakes. In such cases our analysis suggests that if the possibility of value destruction appears significant, potential remedies lie not only in intervening against short sellers, but also in determining why blockholders are unwilling to hold the necessary stake and then appropriately incentivizing them to increase their positions. This should provide important flexibility in balancing the need to prevent the shorts from destroying value against the desire to let them prevent stocks from getting overpriced.

[4] If short selling was unlimited, there would be no equilibrium in pure strategies since the speculator and an informed long-term investor with a good signal would have incentives to engage in a "war of attrition," each trying to out-do the other. See also footnote 11 for papers which document that taking short positions is more expensive and more difficult than taking long positions.

[5] See Section 4 for an analysis of the blockholder's willingness to hold the necessary stake size.

Our analysis also provides a number of new empirical implications. In particular, it implies that short sellers are most likely to destroy value when: (1) long-term shareholders' stakes are inadequate relative to the expected amount of short selling; (2) short selling restrictions are unexpectedly relaxed; (3) the value at risk in a bear raid is relatively small; (4) decision makers behave in a risk-averse fashion; (5) blockholders' information is relatively precise;[6] and (6) the market in the firm's stock is relatively illiquid (allowing the speculator to have a larger relative impact through its trades).

This paper builds on Goldstein and Guembel (2008), who similarly model short sellers manipulating prices downwards to influence managers to take bad decisions and destroy firm value. As in our paper, prices are set by a risk neutral market maker on the basis of net order flows. However, unlike our paper they do not consider how the presence of a long-term investor and the size of his position affects the success of the short seller's strategy. Furthermore, their setting requires that the speculator have a reputation for sometimes being informed, while we show that under certain conditions even a speculator that is known to be uninformed about the firm's future prospects can successfully manipulate in the presence of a feedback effect.[7]

Our paper also builds on Khanna and Sonti (2004), who look at the problem from the side of the informed long-term investors who (like here) may manipulate prices upwards to influence managers to accept good projects and increase firm value. However, they do not consider the effect of a speculator on the trading strategies and success of the long investors' strategy. Attari, Banerjee, and Noe (2006) also model value enhancing price manipulation, though around corporate control events. In their setting, institutional investors may strategically "dump" shares to

[6]This somewhat counterintuitive result is discussed further in Section 6.

[7]The fact that our speculator is uninformed about firm fundamentals may seem to imply that *any* agent could undertake the strategy we derive. However, our speculator does need to have the ability to recognize situations where the possibility of profitable speculation exists. That is, she needs to have some expertise in identifying both firms with the right characteristics and times at which important decisions can be affected by shifts in market prices.

induce relationship investors to buy and subsequently intervene in the firm's management. As in Khanna and Sonti (2004) and the present paper, the institutional holders' actions are motivated both by trading profits and by the desire to protect the value of their existing positions.

Earlier papers that model the feedback/amplification effect (though without directly modeling financial markets) include Bernanke and Gertler (1989), which shows that when an initial positive shock to the economy improves firm profits and retained earnings, it allows firms to invest more, further increasing profits and retained earnings and amplifying the upturn. Similarly, Kiyotaki and Moore (1997) show that a positive shock to land prices translates into increased borrowing capacity, allowing for additional investments. Papers that model the feedback effect of financial market prices on fundamentals but without strategic manipulation include Leland (1992), Khanna, Slezak, and Bradley (1994), Dow and Gorton (1997), Subrahmanyam and Titman (2001), and Ozdenoren and Yuan (2008). In many of these papers low price levels are particularly undesirable as they can result in firm or counterparty decisions that make values even lower.

A number of papers in the academic literature support the notion that short sellers bring valuable information to the market and improve market quality (see, e.g., Boehmer, Jones, and Zhang, 2009, Jones and Lamont, 2002, and Asquith and Meulbroek, 1996). These papers find that restrictions on short sellers tend to degrade market quality, and sometimes cause firms to be overvalued.[8] The latter finding is consistent with models of differences in beliefs, such as Miller (1977), but are at variance with Diamond and Verrecchia (1987), which argues that even with constraints on short selling, prices should be unbiased since markets will adjust for the truncated bad news. Duffie, Garleanu, and Pedersen (2002) suggest that over-pricing may simply reflect the presence of lending fees.

[8]Not all evidence is consistent with this argument, however. For example, Kaplan, Moskowitz, and Sensoy (2010) studies an exogenous shock to the supply of lendable shares for a random group of firms and finds that there is very little effect on pricing or market quality.

In our setting, large stockholders play an active stabilizing role to enhance firm value. This is related to Kyle and Vila (1991), Maug (1998), and Kahn and Winton (1998), which model a strategic trader directly taking an action that affects firm value. Other related papers tend to focus either on blockholders who exercise voice by directly intervening in the firms activities (Shleifer and Vishny (1986), Burkart, Gromb, and Panunzi (1997), Faure-Grimaud and Gromb (2004)), or those who use informed trading, also called exit, to improve stock price efficiency and encourage correct actions by managers (Admati and Pfleiderer (2009), Edmans (2008), Edmans and Manso (2008)).

Finally, our analysis is related to the general literature on stock market manipulation. For example, Bagnoli and Lipman (1996) and Vila (1989) both study manipulation involving direct actions such as a takeover bid. Manipulation based on price pressure or information alone has also been studied widely, such as by Jarrow (1992), Allen and Gale (1992), and Chakraborty and Yilmaz (2004).

The paper proceeds as follows. The base model is described in detail in Section 2. The equilibria of the base model are characterized in Section 3. In Section 4 we extend the model to endogenize the agents' initial positions. In Section 5 we show how the removal of the agency problem affects our results. Comparative statics and empirical implications are discussed in Section 6. Section 7 concludes. All proofs can be found in the Appendix.

2. The Base Model

We consider an economy with a single firm that has many indivisible equity shares outstanding. A decision maker (D) must decide whether to accept or reject a relationship with the firm. Firm value is \$1 per share if D rejects the relationship. If D accepts, $d \in (0, 1)$ per share is added to firm value if the future state of nature, $\Theta \in \{B, G\}$, is good ($\Theta = G$), while $d - \epsilon$ per share, where $\epsilon \in (0, d)$, is subtracted from firm value if the state of nature is bad ($\Theta = B$). The ex ante probability of $\Theta = G$ is $\frac{1}{2}$.

We initially assume that the decision maker is risk averse. In our setting it turns out that working with a risk averse agent makes it easier to characterize the conditions under which efficient equilibria can be sustained. We also believe that this best captures the real world situations in which feedback effects are important. For example, in the recent economic crisis decision makers at counterparty firms considering relationships with troubled financial institutions were likely concerned about the personal consequences if such relationships turned bad (such as losing their job during a tough market), and their incentive contracts were unlikely to be designed with such extreme situations in mind. This could make them overly cautious in their dealings with these institutions. We later show that our results are qualitatively similar with a risk neutral decision maker (see Section 5).

There are (potentially) two strategic traders in the model: a risk-neutral, informed long-term shareholder, I, and a risk-neutral, uninformed speculator, S. I enters the game with a long position in the stock equal to $i > 0$, which is consistent with the empirical regulatory that firms often have one or more long-term blockholders. For the base model, we assume that S either never arrives (the "no speculator" case), or arrives with an exogenous position that is long or short s shares with equal probability (the "active speculator" case). The arrival or non-arrival of the speculator is common knowledge, but the magnitude and direction of her position are her private information. We later endogenize the initial position of the speculator by adding an earlier trading round, and verify that the speculator's overall strategy can be profitable (see Section 4).[9] In that section we also consider I's incentive to adjust its stake.

[9]Considering an exogenous position for S is also useful, however, because it captures scenarios where a speculator holds an *effective* position in a firm without owning that firm's stock. For example, the speculator may hold the stock of a competitor or potential acquirer (generally an effective short interest), or a supplier or customer (generally an effective long interest). Kalay and Pant (2008) discuss many such possible "correlated" long and short positions that occur without direct trading in the firm's shares.

In the base model there is a single trading round. Before trading takes place, I receives a signal, $\theta \in \{L, M, H\}$, about the future state of nature, where H is high, M is medium, and L is low. The probability structure of the signals is such that

- $Pr[\theta = H|\Theta = G] = Pr[\theta = L|\Theta = B] = \lambda$,
- $Pr[\theta = H|\Theta = B] = Pr[\theta = L|\Theta = G] = \frac{1}{2} - \lambda$, and
- $Pr[\theta = M] = \frac{1}{2}$.[10]

We assume $\lambda \in (\frac{1}{4}, \frac{1}{2})$ so that the H and L signals are informative in the correct direction (i.e., an H signal implies a higher probability of the good state). No other agents receive any signals regarding the state, and the only way for I to communicate his information to D is through his trading decisions. While our assumption that I receives a private signal but D does not is standard in the feedback literature, all that we require is that I have access to some information that is incremental to D's.

During the trading round, with probability $\frac{1}{2}$ a noise trader places a market order to buy one share and with probability $\frac{1}{2}$ it places an order to sell one share. I can place a market order for any integer quantity. S can place a market order to buy or sell one share, or can choose not to trade. This limit on the speculator's trades captures real life contraints on short selling.[11] It should be noted that limiting the speculator's trades endogenously determines how much I will choose to trade in equilibrium, implying that the interpretation of our results should always be *relative*. So if over some range of I's initial position i the speculator's actions are shown to reduce efficiency, we can say only that this is the case for such i measured *relative* to the

[10] Effectively, then, I is uninformed with probability $\frac{1}{2}$, which is similar to the information structure in Goldstein and Guembel (2008).

[11] Note that it is easy to show that S's willingness to *buy* additional shares would be endogenously limited by the extent of its long position. However, the short sale limit is a binding one – a short speculator would often wish to sell additional shares if she could. A number of empirical papers document that short selling is more expensive and more constrained than taking long positions (see, e.g., D'Avolio, 2002, and Geczy, Musto and Reed, 2002).

existing limits on short sales. Also, for analytical simplicity we do not formally restrict I from any level of short selling, however, it turns out that it is never necessary for I to sell more than two shares in any of the equilibria we derive. Thus, he never needs to sell more than one share short as long as his initial position is at least one share, and there is no effective asymmetry in the two players' ability to short sell.

After the players place their orders, a risk-neutral market maker sees only the net order flow, Q, and then prices the trades at the risk neutral expected value given his inference about I's signal from observing Q. We represent this price as $p(Q)$. We assume that the market maker holds sufficient inventory to satisfy any relevant pattern of trades.

Next, D makes his accept/reject decision (based on any information he can learn from the stock price, given that he knows the game being played). The risk neutral I would like D to accept as long as the signal is H or M, and not if the signal is L. However, we assume that D is risk averse to the extent that he will accept only if his posterior after inferring I's signal from the stock price is that the probability of the good state is at least $\frac{1}{3} + \frac{2}{3}\lambda$.[12] Since D is an individual while the value of a firm is at stake in the decision, we assume his overall utility is negligible relative to that of the risk-neutral shareholders of the firm. Thus, we always measure the efficiency of the decision from the point of view of the shareholders.[13]

After the decision is made, the state of nature and resulting firm value are realized. Finally, all stock positions are closed out – long positions are paid the firm value per share, and short positions must be closed out by paying the firm value per share.

[12]This captures a specific level of risk aversion in a reduced form. Lowering or increasing the required probability that the signal is H would capture changes in the level of risk aversion of the decision maker – all that is required for our qualitative results is a minimum level of risk aversion. We discuss the case of a risk neutral decision maker in Section 5.

[13]We do not consider how any surplus arising from the relationship is divided between the firm and the counterparty on whose behalf D makes the relationship decision. Our measure of efficiency remains valid as long as a positive NPV transaction for the firm does not create losses for the owners of the counterparty.

3. Equilibrium

In the base model, we consider only pure strategy sequential equilibria.[14] We also require that the posterior beliefs of D and the market maker about the probability of the good state be weakly increasing in net order flow (including those order flows that do not occur in equilibrium).[15] Where multiple equilibria may exist, we focus on the most efficient ones.

Given that an M signal is received with the same probability in the good and bad states, it is uninformative. Thus, I's posterior after receiving an M signal is the same as the prior: a $\frac{1}{2}$ probability of the good state. Since $\epsilon > 0$, an acceptance is positive NPV given this posterior. The posterior after observing an H signal, using Bayes' rule, is

$$Pr[\Theta = G|\theta = H] = \frac{Pr[\theta = H|\Theta = G]}{Pr[\theta = H|\Theta = G] + Pr[\theta = H|\Theta = B]} = \frac{\lambda}{\lambda + (\frac{1}{2} - \lambda)} = 2\lambda > \frac{1}{2}.$$

Similarly, the posterior after observing an L signal is

$$Pr[\Theta = G|\theta = L] = \frac{Pr[\theta = L|\Theta = G]}{Pr[\theta = L|\Theta = G] + Pr[\theta = L|\Theta = B]} = \frac{\frac{1}{2} - \lambda}{(\frac{1}{2} - \lambda) + \lambda} = 1 - 2\lambda < \frac{1}{2}.$$

We assume

$$V_L \equiv 1 + (1 - 2\lambda)d - 2\lambda(d - \epsilon) < 1,$$

that is, an acceptance is negative NPV given an L signal. Thus, from I's point of view a fully efficient equilibrium is one in which D always accepts when the signal is H or M, but never when the signal is L.

[14]Mixed strategies are necessary when we extend the model to an earlier trading round to show that it is rational for the speculator to follow the strategy we derive. See Section 4 for details.

[15]This assumption rules out "perverse" equilibria, such as those in which I buys more shares after observing an L signal than after observing an H signal, which would mean that prices would actually *decrease* in net order flow over some range. Such equilibria are possible only because of the discrete nature of our modeling assumptions. These equilibria could also be ruled out by assuming a small carrying cost for I when it acquires additional shares and then eliminating equilibria that fail to satisfy the Intuitive Criterion of Cho and Kreps (1987), but that approach makes the analysis much more complicated with no additional insights.

It is useful to define other values analogously as follows:

$$V_M \equiv 1 + \frac{1}{2}d - \frac{1}{2}(d - \epsilon) = 1 + \frac{1}{2}\epsilon$$

is expected firm value per share if the decision maker accepts when $\theta = M$; and

$$V_H \equiv 1 + 2\lambda d - (1 - 2\lambda)(d - \epsilon)$$

is expected firm value per share if D accepts when $\theta = H$. Finally, note that if an agent's posterior is that there is a $\frac{1}{3}$ chance the signal is H and a $\frac{2}{3}$ chance the signal is M then the posterior probability of the good state is

$$\frac{1}{3}(2\lambda) + \frac{2}{3}\left(\frac{1}{2}\right) = \frac{1}{3} + \frac{2}{3}\lambda.$$

This corresponds to the posterior that we have assumed is necessary for D to accept. We thus define

$$V_P \equiv 1 + \left(\frac{1}{3} + \frac{2}{3}\lambda\right)d - \left(\frac{2}{3} - \frac{2}{3}\lambda\right)(d - \epsilon)$$

as the expected firm value per share with an acceptance given that posterior.

We next define notation for the posterior beliefs of the market maker and D for different possible net order flows. Note that in equilibrium it does not matter whether D observes the net order flow or just the price (the one is as good as the other in terms of inferring signal probabilities), so we assume without loss of generality that he can observe the net order flow. As such, the two agents' posterior beliefs are always equivalent. Let $Q = q_S + q_I + q_N$ denote the net order flow realization given trading quantities of q_S for the speculator (if it arrives), q_I for the informed shareholder, and q_N for the noise trader. Throughout, for each possible equilibrium we also use the notation q_I^H, q_I^M, and q_I^L for I's *equilibrium* signal-contingent trades. We denote the posterior belief about the probability of state G given Q as $\mu(Q)$.

Now consider the necessary characteristics of a fully efficient equilibrium, in which D always accepts after an H or M signal and always rejects after an L. The following requirements are immediate (proofs not in the text are in the Appendix).

Lemma 1. Any fully efficient pure strategy equilibrium must be such that I plays the same strategy after an M or H signal ($q_I^M = q_I^H$), and plays a sufficiently different strategy after an L signal so that no possible resulting order flows from that signal could arise from his equilibrium trade after an M or H signal.

If these conditions are violated, then there must be equilibrium order flows where the efficient decision is not taken. If I plays different pure strategies after H and M signals ($q_I^M \neq q_I^H$), then some order flows could occur only following an M, and D must conclude upon seeing those order flows that the signal could not be H and reject. Similarly, if I plays a strategy after an L signal where the resulting order flow could also follow an M or H, when that order flow occurs either D sometimes accepts after an L (if the relative probability of an H signal is high enough) or sometimes rejects after an M or H.

We next determine when such fully efficient equilibria exist for both the no speculator and the active speculator cases. In the active speculator case the speculator's basic incentive is to trade in the direction of her initial position, ie, to buy if long and sell if short. This is because the main tension in the model is whether D will accept after an M signal, and buying tends to reinforce I's basic strategy of buying to signal that an acceptance is good, while selling tends to work against that strategy. Thus, subject to its optimality, we assume the speculator buys a share if initially long and sells a share if initially short (we show in the proof of Proposition 1 in the Appendix that this behavior is, in fact, incentive compatible and individually rational in all of the equilibria we derive).[16]

For the no speculator case, consider the class of potential equilibria where I trades a quantity $q_I^M = q_I^H = q_I^+$ after an M or H signal, and trades $q_I^L \leq q_I^+ - 3$ after an L signal. The trades need to differ by at least 3 so that an L signal trade with a buy from the noise trader cannot be

[16]Note that it is possible for other strategies to be incentive compatible for the speculator in fully efficient equilibria, including perhaps not trading after arriving long, which yields qualitatively similar results. We choose to focus on the *most* active rational strategy for the speculator as this gives the clearest results.

confused with an M or H signal trade with a sell from the noise trader (consistent with Lemma 1). The possible equilibrium order flows after an M or H signal are $Q \in \{q_I^+ - 1, q_I^+ + 1\}$, which occur with equal probaility from I's perspective (given the noise trader's probabilistic actions). After an L signal they are $Q \in \{q_I^+ - 4, q_I^+ - 2\}$ *if* $q_I^L = q_I^+ - 3$ (or *less* if $q_I^L < q_I^+ - 3$), again with equal probability. This class of equilibria represents all possible pure strategy fully efficient equilibria in the no speculator case given our condition that beliefs must be monotonic in order flow (i.e., $q_I^L \leq q_I^M \leq q_I^H$).

Any order flow that can follow an L signal, i.e., $Q \in \{q_I^+ - 4, q_I^+ - 2\}$ if $q_I^L = q_I^+ - 3$, must result in the belief that the signal was L. Using Bayes' Rule, any order flow that can follow an M or H, ie, $Q \in \{q_I^+ - 1, q_I^+ + 1\}$, must result in the belief that there is a $\frac{1}{3}$ probability that the signal was H, and a $\frac{2}{3}$ probability it was M. To see this, note that I is assumed to receive an M signal with unconditional probability $\frac{1}{2}$, and an H signal with unconditional probability $\frac{1}{4}$ (the state is good with probability $\frac{1}{2}$ leading to an H signal with probability λ, and the state is bad with probability $\frac{1}{2}$ leading to an H signal with probability $\frac{1}{2} - \lambda$, so the unconditional probability of an H signal equals $\frac{1}{2}\lambda + \frac{1}{2}\left(\frac{1}{2} - \lambda\right) = \frac{1}{4}$). Thus, when D believes that I is pooling after M and H signals and he observes a corresponding order flow, he must conclude that the signal was H with probability $\frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{2}} = \frac{1}{3}$.

This posterior leads D to accept. Since the market maker believes that D will accept, and has the same posterior belief about the probability of the good state, he sets the price at $p(Q) = V_P$ for such order flows Q (from above, this value corresponds to the stated belief). However, since after an M signal I knows that the expected per share value is actually V_M if D accepts, he expects to take a trading loss equal to $q_I^+(V_M - V_P)$. After an H signal, he analogously expects a trading gain equal to $q_I^+(V_H - V_P)$.

These trading gains and losses lead to two main effects that make it difficult to sustain fully efficient equilibria. First, following an M signal I may not be willing to suffer these trading losses, so may deviate downward to a smaller trade. This will cause a loss with respect to the value of his initial position, i, since a desirable acceptance is unlikely, but will save (at least some of) the potential trading loss. This type of deviation will be more likely the smaller is his initial position i, i.e., the less I cares about the ultimate firm value. On the other hand, I may want to deviate upward to a larger quantity in order to maximize his trading gains following an H signal. The size of his initial position is less of an issue here since D always accepts at higher order flows (so I need not worry about an inefficient decision if he deviates upward).

To determine when these deviations are profitable, we must specify out of equilibrium beliefs for D and the market maker. For all $Q \leq q_I^+ - 2$ we assume a belief that the signal is L (this is pinned down by our belief monotonicity assumption when $q_I^L = q_I^+ - 3$). The belief at $Q = q_I^+$ is pinned down by our monotonicity assumption at a $\frac{1}{3}$ probability of an H signal and $\frac{2}{3}$ probability of an M signal. Finally, for all $Q \geq q_I^+ + 2$ we assume a belief that the signal is H. Note that these assumed beliefs support each potential equilibrium in this class *as strongly as possible* since they make downward deviations after M signals and upward deviations after H signals as unattractive as possible (these beliefs minimize the probability of acceptance following an M for downward deviations, and minimize potential trading profits following an H for upward deviations). Also note that these beliefs imply that for $Q \geq q_I^+ + 2$, D will accept and the price will be V_H; for $Q = q_I^+$, D will accept and the price will be V_P; and for $Q \leq q_I^+ - 2$, D will reject and the price will be 1.

The structure of this potential equilibrium is illustrated in Figure 1 below, which shows the prescribed trading quantities for the different signals, the possible resulting net order flows at the ends of the arrows (with probabilities along the arrows determined by the noise trader's buying or selling 1 share with equal probability), and the resulting equilibrium (and assumed out of equilibrium) prices as described above. Equilibrium order flows and prices are in bold italics, and out of equilibrium quantities are in normal text.

As noted above, the most relevant potential deviations are upward deviations after an H signal and downward deviations after an M signal. First consider an upward deviation by I after an

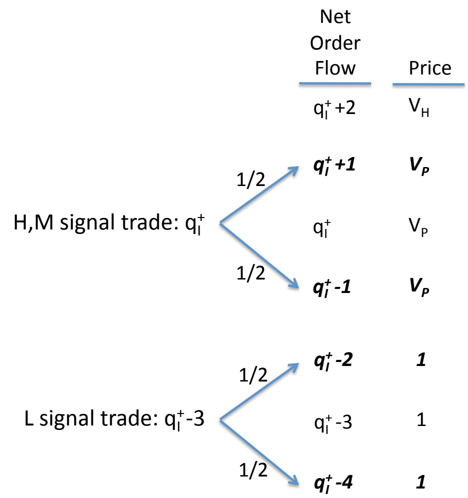

FIGURE 1. Proposed Equilibrium Orders for I, Resulting Net Order Flows, and Prices in the No Speculator Case

H signal in which he places an order of $q_I^+ + 2$ shares instead of q_I^+ shares (see the proof of Proposition 1 in the Appendix for confirmation that the deviations we consider in the text are the most relevant deviations). The resulting potential order flows are $Q \in \{q_I^+ + 1, q_I^+ + 3\}$. This potential deviation is illustrated in Figure 2 below, which lays out the possible order flows and prices after a deviation trade of $q_I^+ + 2$.

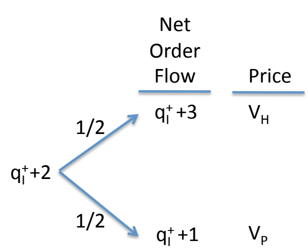

FIGURE 2. Possible Net Order Flows and Prices in the No Speculator Case Following a Deviation Trade of $q_I^+ + 2$ Instead of the Expected q_I^+ After an H Signal

With this deviation, I expects D to accept. With probability $\frac{1}{2}$ the noise trader will sell and the price will be V_P, and with probability $\frac{1}{2}$ the noise trader will buy and the price will be V_H. His expected trading profit is now $\frac{1}{2}(q_I^+ + 2)(V_H - V_P)$. Since he expects an acceptance with certainty (and thus that the value of his existing position to be maximized with either trade), a comparison of this with his expected equilibrium trading profit suffices to test the optimality of the deviation. In particular, the deviation is profitable if $\frac{1}{2}(q_I^+ + 2)(V_H - V_P) > q_I^+(V_H - V_P)$, or, rearranging, if $q_I^+ < 2$. Thus, in the no speculator case, the existence of a fully efficient equilibrium requires that I buy at least 2 shares following an M or H signal, that is, $q_I^+ \geq 2$, so that he will not be able to increase his profits by deviating to a higher quantity after an H signal. This represents the lower bound created by I's trading profits incentive.

Now consider a downward deviation by I after an M signal to a trade of $q_I^+ - 2$. Note from Figure 1 that the possible resulting order flows are $Q \in \{q_I^+ - 3, q_I^+ - 1\}$, with corresponding prices 1 and V_P, respectively. With this deviation, D accepts only with probability $\frac{1}{2}$ in which case the price is V_P (as in the equilibrium), and rejects with probability $\frac{1}{2}$ in which case the price is 1. I's trading loss is therefore $\frac{1}{2}(q_I^+ - 2)(V_M - V_P)$. However, with the change in $D's$ decision, the value of I's initial position must also be considered to determine whether this deviation is profitable. Without the deviation D always accepts, so the value of the initial position is iV_M. When D accepts with probability $\frac{1}{2}$, the value of the position is $i(\frac{1}{2}V_M + \frac{1}{2})$. Thus, the deviation is profitable if $i(\frac{1}{2}V_M + \frac{1}{2}) + \frac{1}{2}(q_I^+ - 2)(V_M - V_P) > iV_M + q_I^+(V_M - V_P)$, or, rearranging, if $i < \frac{(q_I^+ + 2)(V_P - V_M)}{V_M - 1}$. Note that the right-hand side is increasing in q_I^+, which establishes the upper bound on the quantity I is willing to trade with an M signal given his existing position i. Since $q_I^+ \geq +2$ is required from above for this equilibrium to exist, the range of possible existence based on this deviation is $i \geq \frac{4(V_P - V_M)}{V_M - 1}$.

Next consider the active speculator case. To understand the role that the speculator plays, note that her strategy effectively adds noise to the system and allows her to profit from the additional uncertainty created. This has several effects. First of all, it means that I will have to spread his signal-contingent trades wider in order to fully separate his L signal trade from his M and H signal trade. In other words, I will either have to sell more after an L, buy more after an M or H, or both. Second, the additional noise impacts both of the deviation incentives noted above in a way that makes fully efficient equilibria harder to support. In particular, it makes both downward deviations after an M signal and upward deviations after an H signal *more* profitable because the deviations become harder to detect.

To see this, consider the class of equilibria where I trades $q_I = q_I^+$ after an M or H signal (as above), but now trades $q_I = q_I^L \le q_I^+ - 5$ after an L signal to ensure full separation. The difference required for separation increases from three to five shares because the speculator's one-share trades expand the range of "noise" from two to four shares. The possible equilibrium order flows after an M or H signal are now $Q \in \{q_I^+ - 2, q_I^+, q_I^+ + 2\}$, with respective probabilities $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ reflecting the probabilistic actions of the noise trader and speculator. After an L signal they are $Q \in \{q_I^+ - 7, q_I^+ - 5, q_I^+ - 3\}$ if $q_I^L = q_I^+ - 5$ (or *less* if $q_I^L < q_I^+ - 5$). Thus, the L signal is again fully separated as required by Lemma 1. As with the no speculator case above, this class of equilibria is the only possible class of pure strategy fully efficient equilibria in the active speculator case. We specify out of equilibrium beliefs analogously to the no speculator case: the signal is believed to be L for all $Q \le q_I^+ - 3$ and H for all $Q \ge q_I^+ + 3$, while for $Q \in \{q_I^+ - 1, q_I^+ + 1\}$ the monotone beliefs assumption requires the belief that the signal is H with probability $\frac{1}{3}$ and M with probability $\frac{2}{3}$. As above, these beliefs support the equilibrium as strongly as possible. The proposed equilibrium is illustrated in Figure 3 below. Again, equilibrium quantities are in bold italics, and out of equilibrium quantities are in normal text.

Now consider an upward deviation by I to a trade of $q_I^+ + 2$ following an H signal. In the no speculator case, this deviation entailed giving up trading profits $\frac{1}{2}$ of the time, but now, because of the extra noise created by the speculator, I must forego trading profits only $\frac{1}{4}$ of the time for the same increase in trading quantity. See Figure 4 below for an illustration.

This means that expected trading profits are now $\frac{3}{4}(q_I^+ + 2)(V_H - V_P)$. Comparing this with the equilibrium trading profits of $q_I^+(V_H - V_P)$ (again ignoring the value of I's initial position since D always accepts either way), this deviation is profitable if $\frac{3}{4}(q_I^+ + 2)(V_H - V_P) > q_I^+(V_H - V_P)$, or, rearranging, if $q_I^+ < 6$. Thus, whereas with no speculator I had to buy at least 2 shares after an M or H signal to support the equilibrium, with an active speculator that requirement

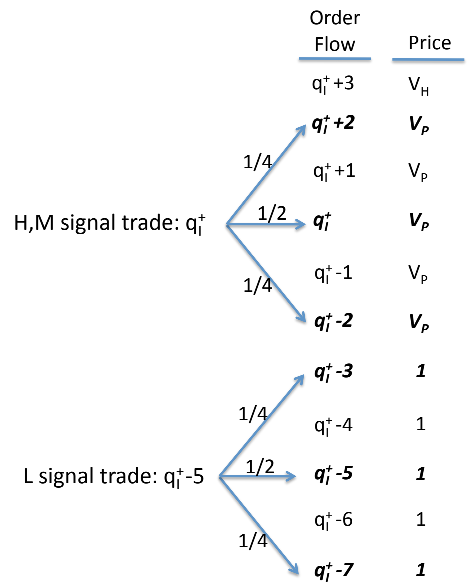

Figure 3. Proposed Equilibrium Orders for I, Resulting Net Order Flows, and Prices in the Active Speculator Case

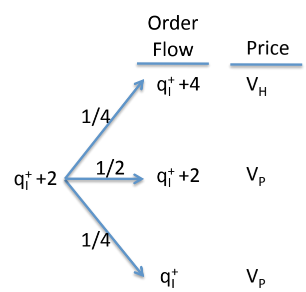

Figure 4. Possible Net Order Flows and Prices in the Active Speculator Case Following a Deviation Trade of $q_I^+ + 2$ Instead of the Expected q_I^+ After an H Signal

triples to 6 shares (i.e., $q_I^+ \geq 6$) because of the increase in his ability to hide the deviation. This illustrates the "multiplier" effect discussed in the introduction.

Finally, consider a downward deviation by I to $q_I^+ - 2$ following an M signal. With no speculator, this deviation resulted in a rejection by D half of the time, but now it does so only $\frac{1}{4}$ of the time. The possible order flows are $Q \in \{q_I^+ - 4, q_I^+ - 2, q_I^+\}$, and with reference to Figure 3 D rejects only at the lowest of the three. The expected payoff to this deviation is therefore $i(\frac{3}{4}V_M + \frac{1}{4}) + \frac{3}{4}(q_I^+ - 2)(V_M - V_P)$. Comparing this to the equilibrium payoff, the deviation is profitable if $i(\frac{3}{4}V_M + \frac{1}{4}) + \frac{3}{4}(q_I^+ - 2)(V_M - V_P) > iV_M + q_I^+(V_M - V_P)$, or, rearranging, if $i < \frac{(q_I^+ + 6)(V_P - V_M)}{V_M - 1}$. As above, the right-hand side is increasing in q_I^+, and since $q_I^+ \geq +6$ is required for this equilibrium to exist because of the multiplier effect, the range of possible existence is $i \geq \frac{12(V_P - V_M)}{V_M - 1}$, or three times that with no speculator.

Verifying the existence of these fully efficient equilibria over the derived ranges also requires showing that I will not deviate either up or down after an L signal, and will not deviate *downward* after an H signal or *upward* after an M signal. With respect to the L signal, note that I makes no trading profit or loss in equilibrium (the price is always correctly 1), and the value of his position i is maximized by non-acceptance since an acceptance is negative NPV. The only possibility for a trading profit with an L would be if I could *sell* some quantity for "too high" of a price and cause an inefficient acceptance some of the time (buying and having D accept is never optimal because he would be buying at too high of a price, leading to a trading loss). But this is impossible given the results above since a sale of 1 share would result in a maximum order flow of $Q = 0$ in the no speculator case and $Q = +1$ in the active speculator case, which is never sufficient for acceptance given $q_I^+ \geq +2$ with no speculator and $q_I^+ \geq +6$ with an active speculator. With respect to the H signal, note that deviating down will reduce the value of I's initial position (D sometimes rejects) while also reducing his trading profits (there is no profit when D rejects). Similarly, after an M signal an upward deviation would leave the value of the initial position unchanged, but increase the trading loss since the price would sometimes be V_H.

We have the following result.

Proposition 1. *In the no speculator case a fully efficient pure strategy equilibrium exists for all* $i > i^{*N} = \frac{4(V_P - V_M)}{V_M - 1}$*, and no such equilibria exist otherwise. In the active speculator case a fully efficient pure strategy equilibrium exists for all* $i > i^{*S} = \frac{12(V_P - V_M)}{V_M - 1}$*, and no such equilibria exist otherwise. Finally, we clearly have* $i^{*S} > i^{*N}$.

This result implies that there is a large range of the informed shareholder's initial position i for which no fully efficient equilibria exist with an active speculator, but do exist without (which is the "efficiency gap" discussed in the introduction).[17] Thus, the actions of the speculator are likely to reduce efficiency in this region. This occurs because the presence of the speculator means that I must buy more in equilibrium in order to ensure that D will accept, which does not create problems with an H signal but does with an M. With an M signal, I does not buy more shares because he would have to incur a larger trading loss and for this range of existing positions the trading loss dominates the gain from ensuring the right decision.

However, in the range where full efficiency exists, whether or not there is an active speculator has no impact. It is straightforward to show that, while an active trading strategy in a fully efficient equilibrium can be incentive compatible for the speculator, it will not generate any profits. It will be incentive compatible because, from the speculator's perspective, all of her trades are at zero profit or zero loss. The only other possible source of profit is an increase in the value of her initial position, but in a fully efficient equilibrium her presence does not affect overall firm value, so no profit occurs. To determine whether the speculator will ever profit from actively trading, we need to determine what type of equilibria may exist over ranges *without* fully efficient equilibria, and whether any such equilibria support profitable speculation.

We continue the strategy of first determining the most efficient possible equilibrium, and then checking for its existence. We assume for the active speculator case that the speculator optimally buys if initially long and sells if initially short. The conditions under which this is optimal for the derived equilibria are given in Proposition 3 below (and proven in the Appendix). One possible equilibrium (which exists whenever $i > 0$) is a fully separating equilibrium where I trades a large positive amount after an H signal, and trades any amount after an M or L that

[17]Note that it is straightforward to show that the entire range of the efficiency gap, $i \in [i^{*N}, i^{*S}]$, always involves positions i in excess of one share (i.e., $i^{*N} > 1$ always holds), which is the technical minimum allowed since we have assumed indivisible shares.

separates them from the trade following an H.[18] However, there are some intermediate equilibria that are both more efficient and allow for potential profits for the speculator. In particular, we characterize the existence of pure strategy "partial pooling" equilibria in which D always accepts after an H, never accepts after an L, and sometimes accepts and sometimes rejects after an M.

For now assume again that I is always willing to separate himself after an L signal to ensure a rejection (which is verified in the proof of Proposition 2). In order to have an equilibrium where D sometimes accepts after an M, I's trades after M and H signals must be separated by a multiple of 2, i.e., after an M he must trade either 2 or 4 shares fewer than after an H (the monotone beliefs assumption requires that I trade fewer shares after an M than after an H). If they were not separated by multiples of 2, then the resulting order flows could never coincide (the strategy would always result in odd net order flows after one signal, and even net order flows after the other). Furthermore, the maximum combined trade of the noise trader and S is 2 shares in either direction, so if the M and H trades are more than 6 shares apart, they can never overlap. Analyzing the possible equilibria provides the following result.

Lemma 2. The most efficient possible pure strategy partial pooling equilibrium has: in the active speculator case, an acceptance after an H signal with certainty, an acceptance after an M signal with probability $\frac{1}{4}$, and a rejection after an L signal with certainty; in the no speculator case, an acceptance after an H signal with certainty, an acceptance after an M signal with probability $\frac{1}{2}$, and a rejection after an L signal with certainty.

When the speculator is active, I trades quantities that are either 2 shares or 4 shares apart after M and H signals. Each trade has three possible outcomes depending on whether S and the noise trader trade in the same direction up or down, or cancel each other out. It is more efficient if

[18]This results in acceptance only after an H, so I is indifferent over his equilibrium trading quantity after an M or L. To see this, note that all trades after an M or L are always correctly priced at $p(Q) = 1$ as long as the resulting order flows could not arise from I's equilibrium trade following an H, so there is no trading loss or gain.

their trades are 4 shares apart. To see this, consider a potential equilibrium in which I is expected to buy 5 shares after an H signal, which results in possible net order flows of $Q \in \{+3, +5, +7\}$ with corresponding probabilities $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$. If he buys 3 shares after an M signal, the net order flow possibilities are $Q \in \{+1, +3, +5\}$, again with corresponding probabilities $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$. Thus, at an order flow of $Q = +3$, D will reject (using Bayes' Rule the probability that this order flow resulted from an H signal is $\frac{1}{5}$, which is too low to support acceptance). At an order flow of $Q = +5$, Bayes' Rule implies a belief that the signal is H or M with equal probability. Thus, D will accept and the price is $p(+5) = V_P^+ \equiv \frac{1}{2}V_H + \frac{1}{2}V_M$. Such a potential equilibrium is illustrated below in Figure 5 (note that the L signal has been left out for simplicity). D accepts at order

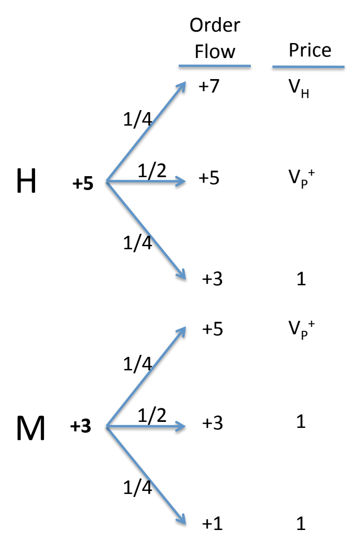

Figure 5. Proposed Equilibrium Orders for I, Resulting Net Order Flows, and Prices for a Partial Pooling Equilibrium with a 2-Share Trading Difference

flows of $Q = +5$ and higher, so overall he accepts with probability $\frac{1}{4}$ after an M signal, but also *rejects* $\frac{1}{4}$ of the time after an H. On the other hand, if I trades $+1$ after an M, the possible order flows are $Q \in \{-1, +1, +3\}$, again with corresponding probabilities $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$. This leads to a belief at $Q = +3$ that the signal was H versus M with probability $\frac{1}{3}$, which is sufficient to ensure acceptance. This possibility is illustrated in Figure 6 below. Here, D will accept at all order flows $Q \geq +3$, implying, again, a $\frac{1}{4}$ chance of acceptance after an M signal, but now

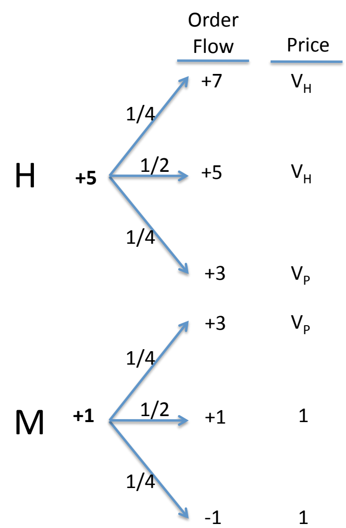

FIGURE 6. Proposed Equilibrium Orders for I, Resulting Net Order Flows, and Prices for a Partial Pooling Equilibrium with a 4-Share Trading Difference

ensuring an acceptance after an H, which is clearly more efficient. Note that in this example, since $q_I^H - 2 = q_I^M + 2 = +3$, the equilibrium prices when D accepts will be $p(+3) = V_P$, $p(+5) = V_H$, and $p(+7) = V_H$. Also note that this analysis extends straightforwardly to any possible base trading quantities – a 4 share trading difference will always be more efficient.

In the no speculator case, since the noise trader's trade is either -1 or +1, I's trades following M and H signals cannot be more than 2 shares apart, else there would be no potential for overlap. For example, if he buys 2 shares after an H, the resulting order flow can be $Q \in \{+1, +3\}$ with probabilities $\{\frac{1}{2}, \frac{1}{2}\}$. Then if he does not trade after an M, the resulting order flow is $Q \in \{-1, +1\}$, again with equal probabilities. Thus, D accepts for all order flows $Q \geq +1$. Here, D accepts with probability $\frac{1}{2}$ after an M and always after an H.

Analyzing such equilibria to determine when they exist provides the following result.

Proposition 2. *A pure strategy equilibrium with partial pooling between H and M signals, with an acceptance for sure following an H signal and with probability $\frac{1}{4}$ following an M signal, exists for all $i \in \left[\frac{2(V_P - V_M)}{V_M - 1}, i^{*S}\right]$ in the active speculator case. A pure strategy partial pooling equilibrium* *with an acceptance for sure following an H signal and with probability* $\frac{1}{2}$ *following an M signal, exists for all* $i \leq i^{*N}$ *in the no speculator case.*

It turns out that these "partial pooling" equilibria give the speculator an opportunity to profit from manipulation. Since S is uninformed, in order to profit she must be able to affect the firm's real value. In the equilibrium described in the above result, the speculator knows that if she sells and the signal is M, D will reject. However, if she buys, D will accept if the noise trader also buys. This wedge gives her the incentive to trade in the direction of her original position since she can cause an inefficient rejection (if she is short and sells) or make an efficient acceptance more likely (if she is long and buys), leading to an increase in the value of her initial position.

However, the trade itself will take place at a loss. Consider what can happen when the speculator sells. On the one hand the signal may be M or L, so D will reject and the price will be $p(Q) = 1$, which is the correct price. On the other hand, the signal may be H. If the noise trader buys, this offsets S's sell trade, the order flow is $Q = q_I^H$, and the price is, correctly, $p(q_I^H) = V_H$. If the noise trader sells, this reinforces S's trade and we have $Q = q_I^H - 2$ and $p(q_I^H - 2) = V_P$ which is *too low* from S's perspective since she knows that such a net order flow can only result after an H signal. Thus, S is being forced to sell at too low of a price and faces an expected trading loss. A similar argument shows that if S buys she will either do so at zero trading profit, or a trading loss due to buying at too high of a price, V_P, when in fact S knows the order flow must have come from an M signal.

This implies that in order for active speculation to be incentive compatible (and profitable), the speculator will have to have a sufficiently large initial position so that the gain on that position will overcome the potential loss from her trade. In fact, we have the following result.

Proposition 3. *In all of the pure strategy partial pooling equilibria derived above for the active speculator case, the speculator's equilibrium strategy is incentive compatible and individually rational as long as her long/short initial position* s *is at least of magnitude* $s^* = \frac{2(V_P - V_M)}{V_M - 1}$.

This result implies that our above assumption about S's actions in the partial pooling equilibria amounts to the assumption that $s \geq s^*$. This confirms that active speculation can be profitable for a speculator that has accumulated a "secret" long or short position in the stock (or an effective position based on correlated instruments such as the stock of a competitor, supplier, customer, counterparty, etc.). Note that the position size required to support active speculation in this class of equilibria is equal to one half of the cutoff below which no fully efficient equilibria exist in the no speculator case. Thus, it is small relative to the initial positions for I that we are focusing on in the region of interest.

The results thus far imply that a speculator's presence can reduce efficiency by causing an inefficient rejection by D. An informed long-term shareholder can prevent this loss, but at an endogenous cost that cannot be justified when his own initial position is not sufficiently large. These results are summarized in Figure 7 below, which plots the corresponding "most efficient" equilibrium as a function of I's initial position i.

The rightward arrow in each panel of the figure represents increasing values of I's initial position, i. The labeled values correspond to the thresholds from propositions 1 and 2. For values of i above $\frac{12(V_P - V_M)}{V_M - 1}$, a fully efficient equilibrium exists with or without the speculator. For values of i from $\frac{4(V_P - V_M)}{V_M - 1}$, or one-third of that level, to $\frac{12(V_P - V_M)}{V_M - 1}$ a fully efficient equilibrium exists without the speculator, while the most efficient equilibrium with the speculator is the partial pooling equilibrium in which D accepts $\frac{1}{4}$ of the time after an M signal. Thus, in this range an active speculator can make profits if its initial position is at least $\frac{2(V_P - V_M)}{V_M - 1}$, and its trading activity can result in significant value loss for the firm.

When i is between $\frac{2(V_P - V_M)}{V_M - 1}$ and $\frac{4(V_P - V_M)}{V_M - 1}$, the most efficient equilibrium both with and without the speculator is a partial pooling equilibrium, but again the best equilibrium with an active speculator is less efficient. Finally, for smaller i the existence of a partial pooling equilibrium is not guaranteed. Thus, in this region we cannot guarantee the existence of an equilibrium with

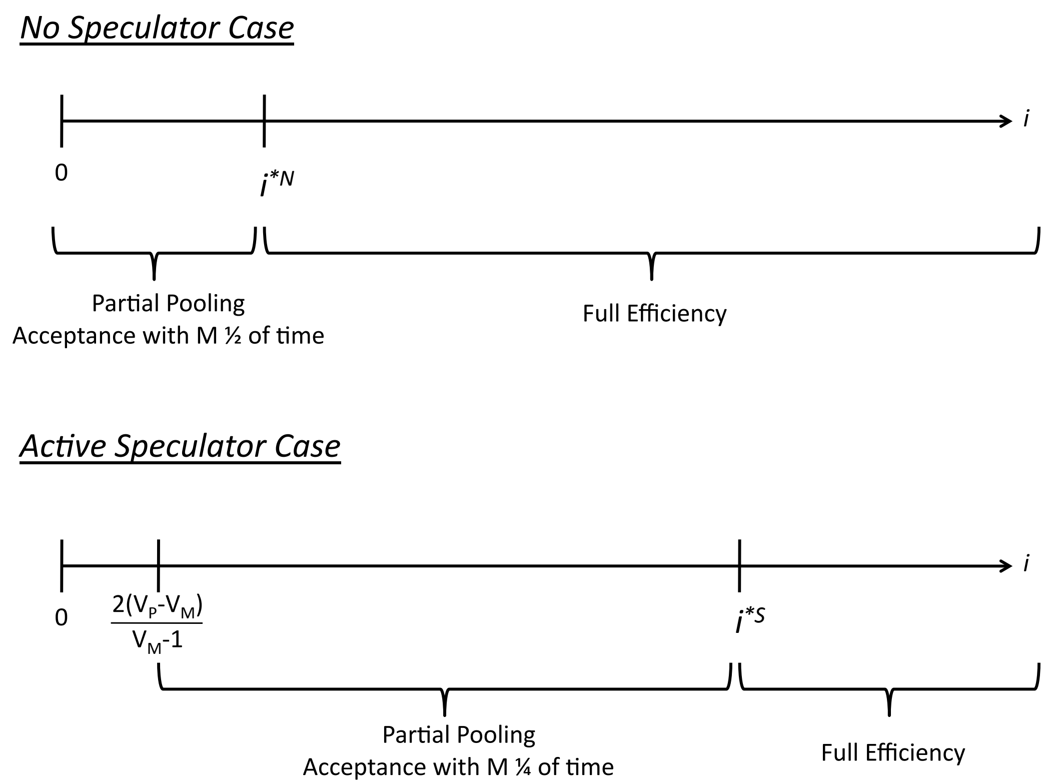

Figure 7. Equilibrium Map

profitable speculation (a separating equilibrium certainly exists, in which case the speculator's actions do not affect efficiency, so S is at best indifferent to trading).

Recall that these results are best interpreted in relative terms since we have restricted the speculator to trading at most one share. In particular, the results indicate that uninformed speculation can (profitably) reduce firm value if informed long-term shareholders' stakes are not large enough *relative* to the existing level of binding short sale constraints.

Clearly, the lower are the thresholds illustrated in Figure 7, the more likely it is that blockholders already hold sufficient stakes and successful bear raids will be prevented. An important question then is how these thresholds vary with the importance of the "at risk" decisions in a bear raid. In this vein, consider the effect of an increase in ϵ. This increases the expected value of the relationship given both H and M signals, but in particular following M signals (the derivative of V_M with respect to ϵ is $\frac{1}{2}$, while the derivative of V_H with respect to ϵ is $(1-2\lambda) < \frac{1}{2}$). Thus, an increase in ϵ can be interpreted as an increase in the importance of the decision following an M signal, and therefore as a measure of how important it is to prevent a successful bear raid. We have the following comparative static.

Proposition 4. *The thresholds i^{*N} and i^{*S} are decreasing in ϵ.*

An increase in ϵ increases the blockholder's incentive to prevent bear raids, making his trading profit motive less important relative to the value loss if the shorts succeed. In addition, the trading losses he must suffer *decrease* because V_M is increasing faster than V_H, and thus the spread between the pooled price V_P and the expected value V_M is decreasing. When combined, these two (reinforcing) effects imply that the multiplier is smaller, reducing the size of the block needed and making it easier to prevent bear raids. Thus, blockholders are more likely to prevent bear raids precisely when they are likely to be most damaging.

4. The Speculator's Overall Profits and the Size of the Investor's Stake

In this section we extend the base model and analyze both whether an uninformed speculator can accumulate a sufficient initial position in an earlier trading round (hereafter the "first round") to make active speculation profitable, and whether the informed investor will have an incentive to adjust his stake size during that round to prevent a bear raid. As noted previously, S's trades in the base model are executed at an expected loss, so it is necessary that she be able to arrive at that stage (hereafter the "final round") with a sufficiently large (and secret) position. Here we show that the accumulation of such an ex ante position during the first round can be profitable, but only if I does not commit to a large enough block to ensure full efficiency. We also show that I may have the incentive to make that commitment in some circumstances.

To capture this we assume that the noise trader buys 1 share with probability $\frac{1}{2}$ and sells 1 shares with probability $\frac{1}{2}$ in the first round. At the time of this round we assume that I has not yet received his signal about the future state Θ. We also assume that I enters the first round with an initial position in the stock of $\hat{\imath} \geq 1$, and plays a pure strategy of trading q_I^1 shares. Consistent with I being a blockholder, we assume his final position at the end of the first round (i.e., his position upon entering the final round), $i = \hat{\imath} + q_I^1$, can be observed by all players before the start of the final round.[19] The speculator is assumed to arrive at the first round with no position in the stock and can buy or sell one share, or choose not to trade. The market maker observes net order flow and sets the price at the risk neutral expected value (with full knowledge of the potential implications of that order flow for the final round).

In the first round the speculator must play a mixed strategy to make trading profits, since otherwise she will always arrive in the final round with a known position. In particular, we show that it is an equilibrium strategy for S to buy one share with probability $\frac{1}{2}$ and sell one share with probability $\frac{1}{2}$. This obviously corresponds to the base model's assumptions by equating $s = 1$, s being the magnitude of the speculator's position upon entering the final round.

If the speculator mixes in this way between buying and selling, her position will be hidden (secret) only $\frac{1}{2}$ of the time - when the noise trader's trade goes in the opposite direction. When their trades are reinforcing, the speculator's trade will be revealed given that I plays the pure strategy described above. Thus, in order to prove that the mixed strategy for S is part of an overall equilibrium, we must consider what happens in the final round when S's position from the first round has been revealed. We show that in such a case, a pure strategy equilibrium exists in which S trades a single share in the same direction as her first round trade. That is, she buys one share if she went long in the first round and sells one share if she went short in the first round.

Lemma 3. If S's position on entering the base model has been revealed, there exists a pure strategy equilibrium in which S trades in the direction of her position, and I trades the same

[19]Current regulations require large shareholders to disclose changes in their positions, albeit with some lag.

quantities as those in the full efficiency and partial pooling equilibria described above for the no speculator case. Furthermore, S's trades in this equilibrium occur at zero profit.

Essentially, if the speculator arrives with a known position and is expected to play a pure strategy in the final round, the market maker and decision maker ignore her effect on the net order flow, and the equilibrium is essentially the same as with no speculator. Since S's trades do not affect any outcomes, they are priced correctly (from her perspective) and entail no trading loss. From here forward we assume these equilibria form the subgame following outcomes where S's first round trade is revealed by a reinforcing noise trade, whereas the most efficient available full efficiency and partial pooling equilibria derived in the previous section form the subgame following outcomes where S's first round trade is hidden (i.e., the noise trader trades in the opposite direction in that round).

Some additional assumptions about the base model's subgame equilibria are needed to complete the analysis. In particular, we need assumptions on what equilibria prevail in the final round if I enters that round with a short position, or enters with a long position smaller than s^* and the speculator's first round trade is hidden. For the latter situation, we assume a separating equilibrium prevails in which the decision maker always accepts after an H signal but never after an M or L. For the case where I arrives short in the final round, we assume the decision maker always rejects (note that the only informed agent in this case has the incentive to destroy value). It is straightforward to show the existence of these equilibria in the respective cases. Given these assumptions about the subgame, we have the following result.

Proposition 5. *If $s^* < 1$, it is incentive compatible and individually rational for S to buy one share with probability $\frac{1}{2}$ and sell one share with probability $\frac{1}{2}$ in the first round. Furthermore, if $s^* \leq \hat{\imath} + q_I^1 \leq i^{*S}$ also holds, S makes a positive overall expected profit.*

This result shows that the speculator's ability to profit by trading in the first round is guaranteed as long as she can secretly trade more shares than are required to satisfy her incentive compatibility constraint in the final round, and $\hat{i}$ and q_I^1 are such that the subgame equilibrium for the final round is a partial pooling equilibrium when her first round trade is hidden. Any additional amount she can trade, $1-s^*$, represents profit. The first round trade is profitable because, in states where her trade is hidden, the trade is priced at an average of the expected value of the firm with a long versus short speculator in the final round. There is a gap between these values since, in the partial pooling equilibrium, the speculator will either induce an increase in firm value on average (if long) or induce a decrease (if short). The speculator plays off this gap, capturing first round expected trading profits that exceed the expected final round losses. It is also worth noting that $s^* < 1$ holds for many relevant parameterizations of the model.

Now consider I's potential pure strategies in this trading round. For tractability, from here forward we assume $s^* \leq 1$ (so that that the speculator's strategy derived above is incentive compatible). Note that if $\hat{i} > i^{*S}$, the full efficiency equilibrium is expected to prevail even if I does not trade at this stage, so the value of the firm is already maximized and there is likely no reason to trade (this is verified in the proof of Proposition 6 below). If not, I may have an incentive to increase his holdings above the threshold to prevent manipulation by the speculator and maximize the value of his existing stake. The question then is whether he is willing to do this given the effect it will have on his present and future trading profits. We now show that a pure strategy equilibrium exists in which I trades at least $i^{*S} - \hat{i}$ shares and prevents successful bear raids in the final round.

Proposition 6. *Given $\hat{i} \geq 1$, an equilibrium of the first round exists in which I undertakes a pure strategy of buying at least $i^{*S} - \hat{i}$ shares if $\hat{i} < i^{*S}$, and does not trade otherwise.*

Thus, given that I starts with a positive but inadequate initial holding, he finds it worthwhile to increase his stake size in the first round to a level where he commits to prevent successful bear raids in the final round. To see why, note that in a fully efficient equilibrium the ex ante expected trading profit for I is zero. This is because I's expected trading losses with an M signal exactly offset his expected gains with an H signal since the pooled price V_P is formed using the ex ante expected probabilities of the M and H signals. On the other hand, I has a positive ex ante expected trading profit in a pooling equilibrium since he trades a larger quantity with an H signal than with an M signal. Thus, I will experience a reduction in expected trading profits if he increases the size of his stake from the range where a partial pooling equilibrium is expected to the range where a fully efficient equilibrium is expected. Furthermore, he might be able to achieve some trading profits today if his future stake and actions are uncertain. Given this, his objective function for first round trading trades off the loss of potential trading profits across the two rounds against the increase in the value of his existing stake, $\hat{i}$. The result shows that the latter effect is always dominant if I enters the first stage as a shareholder.

However, our model can also be seen as a reduced form version of one with stochastic timing of an imminent decision that can be manipulated (i.e., uncertain timing of the final round). In such a situation, the blockholder may need to hold the required block for an indefinite period. In the real world there are many factors affecting blockholders' willingness to hold large stakes over time that could be relevant to such situations, including diversification concerns, capital constraints, regulatory constraints, etc. Such factors may be expected to work against blockholders' incentives to commit to hold large enough stakes.

To see the potential importance of these factors, consider a slight modification of the model in which the initial stake $\hat{i}$ also represents an optimal stake size for the blockholder in the absence of the situation described in the base model (ie, if the probability of arrival for the final round approaches zero). This would be determined by factors outside the model, such as, for example, diversification motives. Now assume that deviating from that stake size between the first and final rounds to a holding of size i entails diversification costs $c(i - \hat{i})^2$, where $c > 0$. Assuming $\hat{i} < i^{*S}$, this has the effect of reducing the equilibrium payoff to the pure strategy described in Proposition 6, and therefore increasing the relative payoff to deviations that leave the final stake closer to $\hat{i}$. This gives the following result.

Proposition 7. *Assume $\hat{i} < i^{*S}$. Then there exists a threshold level of c, c^*, such that the pure strategy equilibrium described in Proposition 6 exists for $c < c^*$ and does not exist for $c > c^*$.*

An additional important question is how the underlying primitives of the model affect the likelihood of an efficient outcome. As noted in Proposition 4, the threshold i^{*S} is decreasing in ϵ. This means that diversification costs become less important as ϵ increases, and we have the following corollary result.

Proposition 8. *The threshold c^* is (weakly) decreasing in ϵ.*

This comparative static has important real world implications. An increase in ϵ has the effect of making decisions following M signals more important since their value impact has increased. Thus, this comparative static implies that blockholders should be more willing to commit themselves to preventing bear raids exactly when such an action has greater value impact. This is not only because of the greater value impact of the at risk decisions, but also because an increase in ϵ tends to dampen the multiplier effect (for reasons noted in Proposition 4), so a smaller stake suffices for full efficiency in the final trading round.

5. The Case of No Agency Problem

In this section we investigate how the base model's results would change in the absence of the agency problem between I and D, that is, if D were willing to accept even if the signal were known to be M. In this setting, a fully separating equilibrium would be fully efficient. However, it generally does not exist. Since D will accept whenever the signal is perceived to be M or higher, after an H signal I will always want to "pool" the H and M signals to some degree to make trading profits without risking an inefficient rejection. As such any feasible fully efficient equilibrium must involve at least some pooling between M and H signals. The important condition for full efficiency is then that I trade a sufficiently low quantity after an L signal so as to completely separate from the M and H signals.

One possible type of fully efficient equilibrium will be the same as the set of fully efficient equilibria derived above. In fact, it is straightforward to show that any fully efficient equilibrium that exists *with* the agency problem also exists *without* it. However, taking away the agency problem makes some additional fully efficient equilibria possible – those where the M and H signals are partially pooled, while the L signals are completely separated. Analyzing such possible equilibria yields the following result.

Proposition 9. *If there is no agency problem between I and D, a fully efficient equilibrium exists in the active speculator case for all $i > i^{*\hat{S}}$, where $i^{*\hat{S}} \leq i^{*S}$, and the inequality is strict for sufficiently small ϵ.*

This result confirms that the agency problem tends to make efficiency more difficult to achieve, and creates additional room for harmful speculation by short sellers. However, taking away the agency problem does not completely solve the efficiency problem. Since I will always want to pool with the M signal after receiving an H, trading positive quantities is still costly for him after receiving an M – some trading losses will always be necessary if I is expected to buy shares after an M. One potential solution would then be for I to trade a very small quantity or not trade at all after an M signal. For example, a possible fully efficient equilibrium would be for I to buy 2 shares after an H signal, not trade after an M signal, and sell 5 shares after an L signal. However, this creates a perverse incentive for I after receiving an L signal. If I's initial position i is small, then after receiving an L he will perceive that if he sells fewer shares, the L signal will sometimes be confused with an M signal, and D may inefficiently accept. This gives him an expected trading profit since he sells at "too high" of a price. Thus, a sufficiently large position i is required to ensure that he will sell 5 shares after an L. With the agency problem, this was never an issue because D would not accept if the signal were perceived to be M. Thus, removing the agency problem actually makes the model more difficult to solve.

The reason the inequality in the result is weak unless ϵ is sufficiently small relates to the above mentioned incentive for I to deviate after an L signal. As ϵ gets larger, the NPV of an acceptance with an L signal approaches zero. Thus, the decline in the value of I's initial position from an inefficient acceptance gets smaller. However, the trading profits do not shrink as ϵ rises, which means the incentive to deviate can become very strong, so that deviation cannot be prevented with an initial position smaller than i^{*S}.

6. Empirical Implications

Our model provides a number of new empirical implications. Most importantly, it implies that value-destroying speculation should be more likely (in terms of both frequency and success) when the holdings of informed, long-term shareholders are small relative to the feasible extent of short selling. It is also more likely when a significant agency problem exists between shareholders and decision makers (decision makers are more risk averse with respect to the firm's dealings), and when the decisions at risk are less important. Finally, in our specification manipulation causes inefficient decisions only for those that are not expected to have the highest impact – i.e., those with M rather than H signals.

We can derive some additional comparative statics from the thresholds in Proposition 1:

Proposition 10. *The thresholds i^{*N} and i^{*S} are increasing in λ and d. However, if $\epsilon = \kappa d$ for some proportion $\kappa < 1$, then the thresholds are independent of d.*

The result with respect to λ implies that the speculator will be more likely to find manipulation profitable if informed shareholders' information is relatively precise (when their signal is, in fact, informative). Intuitively, an increase in λ increases the wedge between the perceived NPV of an acceptance with an H versus M signal - driving V_H up while leaving V_M unchanged. This raises the price V_P without increasing the incentive for I to ensure an acceptance after an M signal. As a result, it is harder to get him to pool – i.e., pooling requires a larger initial position i. This has direct empirical implications about which situations are more amenable to manipulation.

The result with respect to d is similar. Increasing d without changing ϵ makes an acceptance more profitable overall, which interacts with the better information under an H signal to make downward deviation more likely for I after an M. Thus, decisions that ex ante look more profitable are more likely to encourage speculators to manipulate prices. On the other hand, if ϵ and d are held in strict proportion, a change in decision "scale" (an increase in d and ϵ in lockstep) has no effect on the thresholds. This is because the increased impact of the decision affects I's incentive to ensure an acceptance after an M signal and the trading losses required to do so by the same proportion. Overall, these results imply that profitability matters more than scale in terms of predicting when manipulation is likely.

Going outside the strict confines of the model, we can provide additional predictions with respect to which types of firms and situations are likely to become more vulnerable to attempted manipulation. First, our model implicitly assumes that opportunities depreciate relatively quickly - i.e., if a relationship is not accepted the decision cannot be changed later. The problem could clearly be ameliorated if this were not the case and the value loss were less permanent. Second, stocks with lower liquidity in general are likely more vulnerable for two reasons. For one, this allows the speculator's trades to have a greater price impact, increasing her ability to affect outcomes. For another, the additional liquidity provided by the speculator's trades is more likely to cause I to deviate from a pooling equilibrium – i.e., the speculator's trades will have a greater impact on the informed shareholder's willingness to trade sufficient amounts to counteract the potential speculative attack. Third, an unexpected relaxation of restrictions on short selling could create an imbalance between the longs and the shorts, potentially allowing for successful bear raids.

7. Conclusion

We argue that the existing debate over the costs and benefits of short selling activity has overlooked an important participant in the market, the long term blockholder. If there is concern that short sellers can cause an inferior allocation of resources by manipulating stock prices down, blockholders have a powerful natural incentive to battle the shorts in an attempt to prevent such undesirable outcomes. However, because of the tradeoff between making trading profits on their private information and ensuring that bad decisions are not made, the amount of buying needed by blockholders turns out to be a large multiple of the expected amount of short selling. Since some of this buying may need to be done at unfavorable prices it can lead to significant trading losses. If the blockholder's existing stake is insufficient to justify incurring these losses, then short sellers may succeed in inducing sub-optimal decisions and destroying value. This possibility may allow even uninformed speculators to develop trading strategies that are ex-ante profitable.

This raises the important question of whether blockholders are willing to hold sufficiently large stakes to prevent successful bear raids. We show that they are more likely to do so when the decisions at risk have larger value impact, which also turn out to be the cases where the costs of preventing bear raids are lower. However, short selling abuses arguably seem to exist in practice, implying that blockholders sometimes choose not to hold sufficiently large stakes. Possible reasons include diversification motives; uncertainty about the extent of potential short selling; unexpected or sudden relaxing of market-imposed constraints on short selling through, for instance, non-exchange-traded CDS contracts; or simply because the timing of important decisions is stochastic requiring blockholders to hold large undiversified positions for indefinite periods. This suggests that if the possibility of value destruction is considered signicant, potential remedies may lie not only in restricting short sellers, but also in enhancing the incentives for blockholders to increase their positions. The paper also points out that existing restrictions on short sales, which make it harder for speculators to amass positions, play an important role. This raises an important question as to whether these restrictions are optimal. These and other questions are left for future research.

8. Appendix

Proof of Lemma 1: Given any pure strategy for S, if $q_I^H \neq q_I^M$ then there will be some order flows Q after a trade of q_I^M such that only trades of q_I^M or q_I^L could result in those order flows in equilibrium. Since beliefs $\mu(Q)$ must be consistent with Bayes' rule for any equilibrium order flow, the beliefs must place zero probability on an H signal at such order flows and D will not accept. With respect to I's strategy after an L, if $q_I^H = q_I^M$ while q_I^L is such that the resulting equilibrium order flows could not follow a trade of q_I^M, then all possible equilibrium order flows Q that *can* result after a trade of $q_I^M = q_I^H$ will lead to beliefs $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$, which is just sufficient for acceptance. If instead q_I^L were such that any of the possible resulting order flows could also result from a trade of $q_I^M = q_I^H$, then by Bayes' rule the posterior would have to include some probability of an L signal, implying $\mu(Q) < \frac{1}{3} + \frac{2}{3}\lambda$ so that D would reject. QED

Proof of Proposition 1: The remaining issues not proven in the text are: showing that the speculator's trades are incentive compatible and individually rational in the active speculator case; and showing that the deviations considered in the text are the most relevant deviations. First consider the speculator's trades. Note that given the equilibria under consideration, S's trade cannot affect $D's$ decision following any signal. Then denoting the expected value of the firm in equilibrium as $E(V)$, S's expected payoff is $sE(V)$ no matter the quantity she trades since her trades are at zero expected profit or loss. To see this, note that the expected price of any of her trades is $\frac{3}{4}V_P + \frac{1}{4}$, while the expected value of the firm is $\frac{1}{4}V_H + \frac{1}{2}V_M + \frac{1}{4}$, which are equivalent (to see this, replace V_P with $\frac{1}{3}V_H + \frac{2}{3}V_M$). Since a trade of zero is in the choice set, individual rationality is guaranteed.

We now show that we have focused on the relevant deviations for I in the text. First consider upward deviations after an H signal in the no speculator case. If I deviates up by 3 or more shares, the price is always V_H, so trading profits are eliminated. If I deviates up to $q_I^+ + 1$, the expected trading profit is $\frac{1}{2}(q_I^+ + 1)(V_H - V_P)$, which is lower than that derived for the 2 share deviation in the text. Next consider downward deviations after an M signal in the no speculator case. A downward deviation by 3 or more shares results in rejection by D, so the expected payoff is i. This is preferred to the equilibrium payoff if $i > iV_M + q_I^+(V_M - V_P)$, or, rearranging, if $i < \frac{q_I^+(V_P - V_M)}{V_M - 1}$, which is always harder to satisfy than the condition for the 2 share deviation in the text. A downward deviation by 1 share yields an expected payoff of $i(\frac{1}{2}V_M + \frac{1}{2}) + \frac{1}{2}(q_I^+ - 1)(V_M - V_P)$ since D accepts half of the time, just as with the 2 share deviation. Since the trading quantity is higher, this expected payoff is clearly always lower than that for the 2 share deviation in the text.

For the active speculator case, consider upward deviations after an H signal. An upward deviation by 1 share has D still always accepting and yields an expected trading profit of $\frac{3}{4}(q_I^+ + 1)(V_H - V_P)$, which is clearly inferior to the 2 share deviation. A 3 share deviation again has D always accepting, and an expected trading profit of $\frac{1}{4}(q_I^+ + 3)(V_H - V_P)$, while a 4 share deviation has expected trading profit of $\frac{1}{4}(q_I^+ + 4)(V_H - V_P)$, which is clearly superior. The 2 share deviation profit is even higher if $\frac{3}{4}(q_I^+ + 2) > \frac{1}{4}(q_I^+ + 4)$, which always holds for $q_I^+ > -1$ and thus always holds in the ranges where the equilibria may exist given the analysis in the text. Deviations up by more than 4 shares yield no trading profits.

Now consider deviations downward after an M signal in the active speculator case. Similar to the upward deviations, it is straightforward to show that a 2 share deviation is better than a 1 share deviation, and a 4 share deviation is better than a 3 share deviation (they have the same acceptance probability and lower trading losses). A 4 share deviation has a $\frac{1}{4}$ probability of acceptance, leading to an expected payoff of $i(\frac{1}{4}V_M + \frac{3}{4}) + \frac{1}{4}(q_I^+ - 4)(V_M - V_P)$. Comparing this to the equilibrium payoff in the text, deviation is profitable if $i < \frac{(q_I^+ + \frac{4}{3})(V_P - V_M)}{V_M - 1}$, which is clearly harder to satisfy than the condition for the 2 share deviation in the text. A deviation by 5 or more shares has zero probability of acceptance, and thus expected payoff of i. This is preferred to the equilibrium payoff if $i < \frac{q_I^+(V_P - V_M)}{V_M - 1}$, which is again harder to satisfy than the 2 share deviation condition. QED

Proof of Lemma 2: Conditional on the assumption that D reject after an L, the result follows from the discussion in the text just before and just after the result. To complete the proof, we show that no equilibrium in which D sometimes accepts after an L can be more efficient. First note that in order for D to accept in equilibrium after an L with some probability, D must believe that there is a significant probability that the signal was in fact H – a mixture between just M and L signals cannot result in a sufficiently high posterior belief since an M signal by itself is insufficient. Next note that there cannot exist an equilibrium in which $q_I^M = q_I^H$, D always accepts after such a trade, and q_I^L is such that any of the resulting order flows could also follow a trade of q_I^M. If $q_I^M = q_I^H$ then the posterior at the resulting equilibrium order flows is $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$, the minimum required for acceptance. Thus, at any Q where a trade of q_I^L by I is also possible, the posterior must be such that D rejects.

Thus, given monotone beliefs, any equilibrium with L signals not fully separated from M or H signals must have I trading less with an M than an H. As shown in the text, any such equilibrium has acceptance after an M with at most $\frac{1}{4}$ probability. QED

Proof of Proposition 2: The proof proceeds by construction. First consider an equilibrium in the no speculator case in which $q_I^H = +2$, $q_I^M = 0$, and $q_I^L = -2$. At order flow $Q = +3$, D and the market maker must infer that the signal is H. At order flow $Q = +1$ their posterior is $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$, so we assume D accepts, which results in price V_P. At all equilibrium order flows $Q \leq 0$ there is no chance of an H signal, so D rejects and the price equals one. We assume out of equilibrium beliefs are such that at order flow $Q = 0$ the signal is assumed to be M, that at all $Q \geq +2$ the signal is assumed to be H, and that at all $Q \leq -2$ the signal is assumed to be L.

First note that deviations by I following an L signal are not optimal. The initial position i has its value maximized when D rejects (as always happens in equilibrium), and the only possibility of trading profits would be if I could sell a smaller number of shares and still have D sometimes accept. This is not possible since a sale of one share is not sufficient to ever get D to accept (the maximum resulting order flow is zero). Next note that upward deviations by I after an H signal cannot be optimal. Any such deviation would have D always accepting, as in equilibrium, and would have trading profits of zero since the price would always be V_H, so the equilibrium payoff is preferred.

Now consider downward deviations by I after an H signal. In equilibrium D always accepts after an H, maximizing the value of i, and I has an expected trading profit of $\frac{1}{2}(2)(V_H - V_P)$. A deviation to $+1$ means that D will accept only $\frac{1}{2}$ of the time, and there are no trading profits (the trades are correctly priced at $Q = 0$ and $Q = +2$ given this deviation). A deviation to 0 has no trading profits and D also accepts only $\frac{1}{2}$ of the time, so this cannot be profitable. Similarly, upward deviations by I will not be profitable – D always accepts at price V_H, so that trading profits are eliminated.

Finally, I has no incentive to deviate down after an M signal. In equilibrium D accepts $\frac{1}{2}$ of the time, and there are no trading profits/losses since he is not trading, i.e., the expected payoff is $i(\frac{1}{2} + \frac{1}{2}V_M)$. After a downward deviation D will always reject and there are still no trading losses in equilibrium. Finally, consider an upward deviation after an M. A deviation up to $+1$ cannot be optimal - D still accepts $\frac{1}{2}$ of the time, but now trading losses occur in those states. A deviation to $+2$ has D always accepting – the expected payoff is $iV_M - \frac{1}{2}(2)(V_P - V_M) - \frac{1}{2}(2)(V_H - V_M)$. Comparing this to the equilibrium expected payoff shows that deviation is profitable if $i > \frac{2(V_P - V_M) + 2(V_H - V_M)}{V_M - 1}$, which equals $2i^{*N}$. Thus, this equilibrium exists for all $i \in [0, 2i^{*N}]$, which proves the result for the no speculator case.

Now consider the active speculator case, and an equilibrium in which $q_I^M \geq +1$, $q_I^H = q_I^M + 4$, and $q_I^L = q_I^M - 2$. At the equilibrium order flows we have: if $Q \in \{q_I^M + 4, q_I^M + 6\}$, D accepts and $p(Q) = V_H$; if $Q = q_I^M + 2$ D accepts and $p(Q) = V_P$; if $Q \leq q_I^M$ D rejects and $p(Q) = 1$. For out of equilibrium beliefs we assume that for all $Q \geq q_I^M + 3$ the signal is assumed to be H, for all $Q \leq q_I^M - 3$ the signal is assumed to be L, for $Q = q_I^M + 1$ the signal is assumed to be H with $\frac{1}{3}$ probability and M otherwise, and at $Q = q_I^M - 1$ the signal is assumed to be M or L with equal probability. For the out of equilibrium order flow $Q = q_I^M + 1$ we have specified that D is indifferent; we further specify that it would accept with 50% probability.

Now consider possible deviations. There is no profitable deviation with an L since I cannot sell any quantity and have positive probability of acceptance (since $q_I^M \geq +1$ – see above discussion for no speculator case). I will never optimally deviate upward with an H since all trading profits will be eliminated (D will always accept at price V_H). Consider downward deviations after an H. In equilibrium I has an expected payoff of $iV_H + \frac{1}{4} * (q_I^M + 4)(V_H - V_P)$. A deviation to $q_I^M + 3$ still has D accepting $\frac{3}{4}$ of the time at price V_H, and the remainder of the time there is a 50/50 chance of acceptance at V_P or rejection. Thus, this reduces both the value of i and the expected trading profits. It is straightforward to show that deviations to $q_I^M + 1$ or less are similarly dominated by a deviation to $q_I^M + 2$. With a deviation to $q_I^M + 2$, D either: accepts at $Q = q_I^M + 4$ at price V_H, accepts at $Q = q_I^M + 2$ at price V_P, or rejects at $Q = q_I^M$. The expected payoff is therefore $i(\frac{1}{4} + \frac{3}{4}V_H) + \frac{1}{2}(q_I^M + 2)(V_H - V_P)$. Comparing this to the equilibrium payoff, the deviation is profitable if $i < \frac{q_I^M(V_H - V_P)}{V_H - 1}$.

Next consider downward deviations after an M. I's equilibrium expected payoff is $i(\frac{3}{4} + \frac{1}{4}V_M) - \frac{1}{4}q_I^M(V_P - V_M)$. If he deviates down by 1 share to $q_I^M - 1$, D will accept with some probability only if $Q = q_I^M + 1$, and then with only $\frac{1}{2}$ probability, which yields an expected payoff of $i(\frac{7}{8} + \frac{1}{8}V_M) + \frac{1}{8}(q_I^M - 1)(V_M - V_P)$. Comparing this to the equilibrium payoff the deviation is profitable if $i < \frac{(q_I^M + 1)(V_P - V_M)}{V_M - 1}$. Deviating down by more than 1 share results in D always rejecting, and thus an expected payoff of i, which is preferable to the equilibrium payoff if $i < \frac{q_I^M(V_P - V_M)}{V_M - 1}$, which is clearly harder to satisfy, so the 1 share deviation is the relevant one to consider. Now compare the 1 share downward deviation condition after an M, $i < \frac{(q_I^M + 1)(V_P - V_M)}{V_M - 1}$, to the two share downward deviation condition after an H, $i < \frac{q_I^M(V_H - V_P)}{V_H - 1}$. By replacing the V terms with their algebraic definitions in terms of the model's primitives, it is straightforward to show that the former equals $\frac{2(q_I^M + 1)Y}{3\epsilon}$ and that the latter equals $\frac{q_I^M Y}{3\gamma}$, where $Y \equiv (2d - \epsilon)(2\lambda - \frac{1}{2})$ and $\gamma \equiv (2d - \epsilon)\lambda + \frac{1}{2}(\epsilon - d)$. Consider the ratio $\frac{\gamma}{\epsilon}$. Our assumption that $V_L < 1$ implies

$\epsilon < \frac{d(4\lambda-1)}{2\lambda}$. We have $\frac{\partial \frac{\gamma}{\epsilon}}{\partial \epsilon} = \frac{d(\frac{1}{2}-2\lambda)}{\epsilon^2} < 0$, and plugging for the maximum ϵ we have $\frac{\gamma}{\epsilon} = \frac{1}{2}$, so $\gamma \geq \frac{1}{2}\epsilon$ must always hold. Plugging this minimum γ into the expression for the downward deviation condition following an H yields $\frac{q_I^M Y}{3\frac{1}{2}\epsilon} = \frac{2q_I^M}{3\epsilon}$, so the downward deviation condition following an M is always larger and thus is the relevant downward cutoff for existence of the equilibrium.

Finally consider upward deviations after an M, in particular a deviation to q_I^M+2 (it is straightforward to show this is the relevant deviation by testing the other possibilities as above). This deviation yields an expected payoff of $i(\frac{1}{4}+\frac{3}{4}V_M)-\frac{1}{4}(q_I^M+2)(V_H-V_M)-\frac{1}{2}(q_I^M+2)(V_P-V_M)$. Comparing this to the equilibrium expected payoff the deviation is profitable if $i > \frac{(\frac{2}{3}q_I^M+\frac{5}{3})(V_H-V_M)}{V_M-1}$. Thus, the relevant range of existence for this equilibrium is $i \in [\frac{(q_I^M+1)(V_P-V_M)}{V_M-1}, \frac{(\frac{2}{3}q_I^M+\frac{5}{3})(V_H-V_M)}{V_M-1}]$, or, replacing the V terms with their equivalents in terms of the primitives and simplifying, $i \in [\frac{2(q_I^M+1)Y}{3\epsilon}, \frac{2(2q_I^M+5)Y}{3\epsilon}]$, where $Y \equiv (2d-\epsilon)(2\lambda-\frac{1}{2})$.

Now note that at the minimum q_I^M we specified, $q_I^M = +1$, the lower boundary clearly corresponds to that given in the result. Also, as q_I^M is increased, both the upper and lower boundaries of existence for the equilibrium increase, and the upper boundary is clearly always greater. It is straightforward to show that $i^{*S} = \frac{8Y}{\epsilon}$, so the upper boundary exceeds i^{*S} at a value of $q_I^M = 4$. Finally, note that the new lower boundary lies below the old upper boundary each time q_I^M is increased by 1 (plugging q_I^M+1 into the lower boundary yields $\frac{2(q_I^M+2)Y}{3\epsilon} < \frac{2(2q_I^M+5)Y}{3\epsilon}$), so considering each equilibrium as q_I^M increases by ones from +1 to +4 yields the result. QED

Proof of Proposition 3: Consider the equilibrium derived in the proof of Proposition 2 in which $q_I^M \geq +1$, $q_I^H = q_I^M+4$, and $q_I^L = q_I^M-2$. The speculator enters the trading round with a position of magnitude s. First assume this is a short position, $-s$. Then if the speculator short sells one share as the equilibrium requires, the possible equilibrium order flows are (from his perspective): if $\theta = L$, $Q \in \{q_I^M-4, q_I^M-2\}$ with equal probability (due to the noise trade); if $\theta = M$, $Q \in \{q_I^M-2, q_I^M\}$ with equal probability; and if $\theta = H$, $Q \in \{q_I^M+2, q_I^M+4\}$ with equal probability. Thus, D will never accept after an M or L signal, and the price will always be 1 in those cases. D will always accept after an H and the price is V_H or V_P with equal probability. L, M, and H signals arrive with ex ante unconditional probabilities of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ respectively. Thus, the expected price is $\frac{3}{4} + \frac{1}{8}V_H + \frac{1}{8}V_P$. The expected value of the shares is $\frac{3}{4} + \frac{1}{4}V_H$. The speculator's expected payoff to the equilibrium strategy is therefore $-s(\frac{3}{4} + \frac{1}{4}V_H) - \frac{1}{8}(V_H - V_P)$ (trading losses occur only when D accepts at price V_P after an H).

The only relevant deviation will be to not trade (buying will further reduce the value of s while also causing trading losses). With a deviation to zero, possible order flows are: if $\theta = L$, $Q \in \{q_I^M - 3, q_I^M - 1\}$; if $\theta = M$, $Q \in \{q_I^M - 1, q_I^M + 1\}$; and if $\theta = H$, $Q \in \{q_I^M + 3, q_I^M + 5\}$. The only differences in outcomes are that D now accepts after an M signal $\frac{1}{4}$ of the time at price V_P (noise buys $\frac{1}{2}$ of the time, and then D accepts $\frac{1}{2}$ of the time when that happens) while the price is always V_H after an H signal. The speculator's expected payoff is therefore $-s(\frac{5}{8} + \frac{1}{8}V_M + \frac{1}{4}V_H)$ since S is trading zero. Comparing this to the equilibrium payoff, the deviation is profitable if $s < \frac{V_H - V_P}{V_M - 1} = \frac{2(V_P - V_M)}{V_M - 1}$, which is the expression provided in the result.

For the case with a long position of s, we similarly must check the deviation to no trade. Following similar logic, the equilbrium expected payoff to buying one share is $s(\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H) - \frac{1}{4}(V_P - V_M)$. The expected payoff to not trading is $s(\frac{5}{8} + \frac{1}{8}V_M + \frac{1}{4}V_H)$. Comparing this to the equilibrium payoff, the deviation is profitable if $s < \frac{V_H - V_P}{V_M - 1} = \frac{2(V_P - V_M)}{V_M - 1}$. QED

Proof of Proposition 4: First note that both thresholds are multiples of s^*. Replacing the terms V_P and V_M in $s^* = \frac{2(V_P - V_M)}{V_M - 1}$ with their expressions in terms of the model primitives yields $s^* = 2\left(\frac{1}{3} + \frac{\frac{2}{3}(4\lambda d - 2\lambda\epsilon - d)}{\epsilon}\right)$. Taking the derivative with respect to ϵ yields $-\frac{\frac{4}{3}d(4\lambda - 1)}{\epsilon^2} < 0$. QED

Proof of Lemma 3: The proof is again by construction. First consider the full efficiency equilibrium analog to the no speculator case in which $q_I^M = q_I^H = q_I^+ \geq +2$ and $q_I^L = q_I^+ - 3$. Now assume the speculator arrives long s shares (which is common knowledge) and is prescribed to buy one share. Possible equilibrium order flows are $Q \in \{q_I^+, q_I^+ + 2\}$ following H and M signals depending on whether the noise trader buys or sells. Thus, D accepts at these order flows and the price is V_P. An L signal results in $Q \in \{q_I^+ - 3, q_I^+ - 1\}$, so D rejects for all $Q \leq q_I^+ - 1$ and the price is 1 (out of equilibrium beliefs must place all weight on L in that range). Our monotone beliefs assumption requires that at out of equilibrium node $Q = q_I^+ + 1$, D be indifferent, so we assume he accepts and the price is V_P. Checking the possible deviations by I proceeds as in the proof of Proposition 1 (note that I still cannot deviate to a "sell" quantity after an L that gets D to accept, as selling one share leads to a maximum order flow of $q_I^+ - 1$ if S is long and buys, which is insufficient to get it accepted, and $q_I^+ - 3$ if S is short and sells, which is again insufficient to get D to accept – see below), and it is straightforward to show that I's incentive to deviate downward to $q_I^+ - 2$ after an M again limits the range of existence to $i \geq i^{*N}$. The only remaining deviations to check are deviations by S.

In equilibrium, S's expected payoff is $s(\frac{1}{4} + \frac{1}{2}V_M + \frac{1}{4}V_H)$ (S's trade is always executed at the true expected value from her perspective). If S deviates to zero the M or H signal order flow becomes $Q \in \{q_I^+ - 1, q_I^+ + 1\}$, so D will accept only $\frac{1}{2}$ of the time, reducing the expected payoff to $s(\frac{5}{8} + \frac{1}{4}V_M + \frac{1}{8}V_H)$. If S deviates to -1, D will again accept only $\frac{1}{2}$ of the time, and there will again be no trading profit. Thus, S will not deviate. The proof for the case where S arrives short s shares is analogous.

Next consider the partial pooling equilibrium in which $q_I^H = +2$, $q_I^M = 0$, and $q_I^L = -2$. Now assume the speculator arrives long s shares (which is common knowledge) and is prescribed to buy one share. The equilibrium order flow possibilities are: if the signal is H, $Q \in \{+4, +2\}$; if the signal is M, $Q \in \{+2, 0\}$; if the signal is L, $Q \in \{-2, 0\}$. Thus, D accepts at price V_H at $Q = +4$, accepts at price V_P at $Q = +2$, and rejects for lower Q. Out of equilibrium beliefs are such that D accepts at price V_H for all $Q \geq +3$, while D rejects for any $Q \leq +1$ (the signal is believed to be M or L). Again, checking for deviations by I proceeds as in prior proofs and shows that the equilibrium exists for the entire range of $i \in [0, i^{*N}]$.

Finally, consider deviations by S. S's equilibrium payoff is $s(\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H)$. If she deviates to zero, D always rejects after an M and accepts only $\frac{1}{2}$ of the time after an H, so $S's$ equilibrium payoff is $s(\frac{7}{8} + \frac{1}{8}V_H)$. A deviation to -1 has the same expected firm value per share, but there is a trading loss because D sometimes accepts after an H at price V_P, which is too low. The proof for when S arrives short is analogous. QED

Proof of Proposition 5: For the purposes of this proof we assume $q_I^1 = 0$. This is without loss of generality since, as noted in the text, any pure strategy for I cannot affect the market maker's inference about S's trades, and thus the proof for any $\hat{i}$ with a pure strategy for I of trading $q_I^1 \neq 0$ shares is equivalent to the proof with $q_I^1 = 0$ and $\hat{i}$ set equal to the sum of $\hat{i}$ and q_I^1 in the former case. Furthermore, with this constraint we need only consider cases where $\hat{i} \in [s^*, i^{*S}]$. If $\hat{i} > i^{*S}$ the full efficiency equilibrium will prevail in the final round, and all of the speculator's trades in both rounds will be correctly priced from her perspective and are therefore incentive compatible and individually rational. Furthermore, if $\hat{i} < s^*$, the speculator's trades in the final round will also occur at zero profit or loss given our assumption that the resulting equilibrium is either a separating equilibrium or one in which the decision maker always rejects (it is easy to show that the speculator's trades cannot change any outcome in either of those equilibria). Thus, again, the speculator's trades in both rounds will be incentive compatible and individually rational

From Lemma 3 and the proceeding text we know that if the speculator and noise trader trade in the same direction, the speculator has zero profit/loss overall (both the first and final round trades occur at zero profit/loss). Note that all of the prices derived above for the active speculator case reflect a $\frac{1}{2}$ probability that S will be long vs. short when its position entering the final trading round is unknown. Thus, using these prices it suffices to show that mixing with probability $\frac{1}{2}$ between buying and selling one share in the first trading round is incentive compatible and individually rational for S. If the speculator does not trade, she gets an overall expected payoff of zero (it is easy to show she can never profit from trading against I with no initial position in the base model, so we assume she would not trade again, leading to an overall expected payoff of zero).

First we derive the first round market price assuming that the speculator's trade is not discovered (ie, the noise trader trades in the opposite direction). Recall that in the range of i we need to consider, we have assumed the final round (base model) equilibrium is the relevant partial pooling equilibrium from Proposition 2. Now, given the order flow of 0, the market maker perceives that there is a 50/50 probability of S having gone long or short. If the speculator is short, the expected per share firm value is $\frac{3}{4}+\frac{1}{4}V_H$ (see the proof of Proposition 3). If the speculator is long, the expected per share firm value is $\frac{1}{2}+\frac{1}{4}V_M+\frac{1}{4}V_H$ (again, see the proof of Prosition 3). The overall expected firm value places $\frac{1}{2}$ weight on each, which yields $\frac{5}{8}+\frac{1}{4}V_H+\frac{1}{8}V_M$, so this is the first round price when S's trade is hidden.

S's expected payoff equals the sum of the expected trading profits (losses) from each round. These both equal zero if the first round trade is revealed (the noise trader trades in the same direction). Thus, the overall expected payoff to S if she buys one share can be expressed as $\frac{1}{2}\left[(\frac{1}{4}V_H+\frac{1}{4}V_M+\frac{1}{2}-\frac{5}{8}-\frac{1}{4}V_H-\frac{1}{8}V_M)-\frac{1}{4}(V_P-V_M)\right]$, where the first term in the brackets is the expected first round trading profit, and the second term is the expected final round trading loss. This simplifies to $\frac{1}{2}\left[\frac{1}{8}(V_M-1)-\frac{1}{4}(V_P-V_M)\right]$. Similarly, the overall expected payoff if S sells one share can be expressed as $\frac{1}{2}[-(\frac{3}{4}+\frac{1}{4}V_H-\frac{5}{8}-\frac{1}{4}V_H-\frac{1}{8}V_M)-\frac{1}{8}(V_H-V_P)]$, which simplifies to $\frac{1}{2}\left[\frac{1}{8}(V_M-1)-\frac{1}{8}(V_H-V_P)\right]$. Note that these are equivalent given $V_P=\frac{1}{3}V_H+\frac{2}{3}V_M$. This proves incentive compatibility.

To prove it is individually rational and profitable, we must show these last expressions are positive given $s^*<1$. From the proof of Propostition 4, $s^*=2\left(\frac{1}{3}+\frac{\frac{2}{3}(4\lambda d-2\lambda\epsilon-d)}{\epsilon}\right)$. It is straightforward to show that $s^*<1$ thus implies $\epsilon>\frac{4d(4\lambda-1)}{1+8\lambda}$. Now note that $\frac{1}{2}\left[\frac{1}{8}(V_M-1)-\frac{1}{8}(V_H-V_P)\right]>0$ holds if $(V_M-1)-(V_H-V_P)>0$. Replacing the defined terms with their expressions in terms of the primitives and rearranging yields $d(\frac{2}{3}-\frac{8}{3}\lambda)+\epsilon(\frac{1}{6}+\frac{4}{3}\lambda)$, which is positive if $\epsilon>\frac{4d(4\lambda-1)}{1+8\lambda}$. QED

Proof of Proposition 6: Here we prove the result for an initial stake $\hat{i}=1$. The proofs for larger stakes proceed analogously. First note that our earlier assumption that $V_L<1$ implies

$\epsilon < \frac{d(4\lambda-1)}{2\lambda}$, which impies $s^* > \frac{2}{3}$. This together with the new assumption that $s^* \leq 1$ implies $i^{*S} \in (4, 6]$. Now consider a candidate pure strategy equilibrium in which I buys the minumum number of whole shares q_I^1 such that $\hat{i} + q_I^1 \geq i^{*S}$, and S trades as described in Proposition 5. Note that since I's stake size is revealed between the first and final rounds, at the time of the final round the other players can infer I's actual first round trade, and their inference about S's trade going into the final round will be based on that knowledge (ie, if the realized order flow was equal to I's trade, the speculator's actual trade will be hidden, and if not the speculator's trade will be known).

In the proposed equilibrium, the expected final value of the firm is known to be equal to the expected value based on a fully efficient equilibrium of the final trading round, which we denote as $V^+ \equiv \frac{1}{4}V_H + \frac{1}{2}V_M + \frac{1}{4}$. Thus, all *equilibrium* order flows will have a price of V^+, and I's trade has no trading profit or loss. Furthermore, since I receives an H signal with ex ante probability $\frac{1}{4}$ and an M signal with ex ante probability $\frac{1}{2}$, his overall expected trading profit in any fully efficient equilibrium of the final round as derived in Proposition 1 (letting the common trading quantity after H and M signals be denoted by q_I^+) can be written as $\frac{1}{4}q_I^+(V_H - V_P) + \frac{1}{2}q_I^+(V_M - V_P)$. Plugging in $V_P = \frac{1}{3}V_H + \frac{2}{3}V_M$, this simplifies to zero. Since expected trading profits are zero in both trading rounds, I's expected equilibrium payoff in this proposed equilibrium is simply $\hat{i}V^+ = V^+$, the induced value of his initial stake. Note that this implies I is indifferent across any deviation trades q^d that lead to a final stake $\hat{i} + q^d > i^{*S}$ as long as the expected price is still V^+; any deviation to a different trade in that range that gets a lower price with some probability (note that higher prices cannot be supported in equilibrium since the market maker cannot believe that firm value will be higher than V^+ in expectation) must be profitable. Thus, all possible out of equilibrium order flows resulting from trades q^d such that $\hat{i} + q^d > i^{*S}$ must have price V^+.

Now consider other possible deviations. Given our assumptions about the subgame equilibrium in the final round, a deviation to any trade q^d such that $\hat{i} + q^d \in [2s^*, i^{*S})$ (which must be a buy trade given $\hat{\imath} = 1$ and our assumptions on s^*) must lead to an expected final round equilibrium (from I's perpective) of either full efficiency as derived in Proposition 1 (if S's trade becomes known because the noise trader traded in the same direction in the first round) or a partial pooling equilibrium as derived in Proposition 2 for the active speculator case (if S's trade remains hidden because the noise trader traded in the opposite direction), with equal probability. The expected firm values based on these possibilities are, respectively, V^+ and $V_{PP}^{US} \equiv \frac{1}{4}V_H + \frac{1}{8}V_M + \frac{5}{8}$. Similarly, any deviation to a trade q^d such that $\hat{\imath} + q^d \in [s^*, 2s^*)$ (which is a trade of zero) must lead to an expected final round equilibrium of partial pooling with or without an active speculator as derived in Proposition 2, with equal probability. The expected firm values in these two cases are V_{PP}^{US} and $V_{PP}^{KS} \equiv \frac{1}{4}V_H + \frac{1}{4}V_M + \frac{1}{2}$. A deviation to a trade q^d such that $\hat{\imath} + q^d \in [0, s^*)$, i.e., a trade of -1, must lead to an expected final round equilibrium of partial pooling with a known speculator, or a fully separating equilibrium with equal probability. Expected firm value is V_{PP}^{KS} in the former case and $V_{SE} \equiv \frac{1}{4}V_H + \frac{3}{4}$ in the latter. Finally, consider a deviation to a trade q^d that leaves $\hat{\imath} + q^d \leq 0$. In this case, the expected firm value is 1, since once I's non-positive position becomes known, the decision maker is assumed to always reject.

First consider a deviation to no trade. In this case, I has no current trading profits or losses. It expects that in the final round there will be a partial pooling equilibrium with a known speculator with probability $\frac{1}{2}$, and a partial pooling equilibrium with an active speculator with probability $\frac{1}{2}$. In the latter case I trades 4 more shares after receiving an H signal than after receiving an M. Letting q_I^M denote the trading quantity after receiving an M, I's expected final round trading profit can be written as $\frac{1}{4}(\frac{1}{4}(q_I^M + 4)(V_H - V_P)) + \frac{1}{2}(\frac{1}{4}q_I^M(V_M - V_P))$. Plugging in $V_P = \frac{1}{3}V_H + \frac{2}{3}V_M$, this simplifies to $\frac{1}{6}(V_H - V_M)$. It is straightforward to show that the expected trading profit is the same for the partial pooling equilibrium with a known speculator. Thus, I's expected profit given this deviation and $\hat{\imath} = 1$ is $(\frac{1}{2}V_{PP}^{KS} + +\frac{1}{2}V_{PP}^{US}) + \frac{1}{6}(V_H - V_M)$. Replacing V_{PP}^{KS} and V_{PP}^{US} with their expressions from above, this simplifies to $\frac{1}{6}(V_M - V_H) + \frac{5}{16}(V_M - 1)$. This is less than the equilibrium payoff of V^+ if $1 > \frac{8}{15}\left(\frac{V_H - V_M}{V_M - 1}\right)$. This always holds when $s^* \leq 1$ (as we have assumed) since $s^* = \frac{2(V_P - V_M)}{V_M - 1} = \frac{2}{3}\left(\frac{V_H - V_M}{V_M - 1}\right)$.

Next consider a deviation to buying one share (it is easy to show this is the hardest remaining "buy" deviation to prevent). In this case the equilibrium in the final round is expected to be fully efficient with probability $\frac{1}{2}$ (if the speculator's trade ultimately becomes known) and to be a partial pooling equilibrium with an active speculator with probability $\frac{1}{2}$ (if the speculator's trade remains hidden). This means that expected future trading profits are $\frac{1}{2}(\frac{1}{6}(V_H - V_M))$. We assume that the price at order flows $+3$, $+1$ and -1 (and all prices in between) are set at V^+, which prevents the deviation as strongly as possible (this price can always be chosen for out of equilibrium order flows in this range since there are always consistent beliefs that can be assigned to these order flows – for example, at an order flow of -1 the market maker could believe that I bought one share and both S and the noise trader sold, leading to a fully efficient equilibrium in the final round and an expected firm value of V^+). Current expected trading profits are therefore $(\frac{1}{2}V^+ + \frac{1}{2}V_{PP}^{US}) - V^+$. The overall expected payoff to this deviation is therefore $\hat{\imath}(\frac{1}{2}V^+ + \frac{1}{2}V_{PP}^{US}) + \frac{1}{12}(V_H - V_M) + (\frac{1}{2}V^+ + \frac{1}{2}V_{PP}^{US}) - V^+$, which, given $\hat{\imath} = 1$, can be rewritten as $V_{PP}^{US} + \frac{1}{12}(V_H - V_M)$. This is less than the equilibrium payoff of V^+ if $1 > \frac{2}{9}\left(\frac{V_H - V_M}{V_M - 1}\right)$. As above, this always holds when $s^* \leq 1$.

Now consider a deviation to a sale of 1 share. The expected equilibrium of the final round will be partial pooling with a known speculator with probability $\frac{1}{2}$ (in which case I has expected future trading profits of $\frac{1}{6}(V_H - V_M)$) and a separating equilibrium with probability $\frac{1}{2}$ (if the speculator's trade is hidden, and in which case there are no expected future trading profits for I). We assumed above that the prices at order flows $+1$ and -1 are V^+. For an order flow of -3 we assume a price of 1. Thus, the expected payoff to the deviation is $\hat{\imath}(\frac{1}{2}V_{PP}^{KS} + \frac{1}{2}V_{SE}) + (\frac{3}{4}V^+ + \frac{1}{4} - (\frac{1}{2}V_{PP}^{KS} + \frac{1}{2}V_{SE})) + \frac{1}{12}(V_H - V_M)$. Using $\hat{\imath} = 1$ and replacing V_{PP}^{KS} and V_{SE} with their definitions from above, this can be simplified to $\frac{3}{4}V^+ + \frac{1}{4} + \frac{1}{12}(V_H - V_M)$. This is less than the equilibrium payoff of V^+ if $1 > \frac{1}{9}\left(\frac{V_H - V_M}{V_M - 1}\right)$. As above, this always holds when $s^* \leq 1$.

Finally, consider a deviation to a sale of 3 shares (it is straightforward to show this is the hardest remaining sale deviation to prevent). Since I will be short in the final round, the decision maker will always reject and firm value is 1. We assumed above that the price at order flow -1 is V^+ and at order flow -3 it is 1. We further assume that the price at all lower order flows is 1. The expected payoff to the deviation is therefore $\hat{\imath}(1) + 3(\frac{1}{4}V^+ + \frac{3}{4} - 1)$. Given $\hat{\imath} = 1$, this becomes $\frac{1}{4} + \frac{3}{4}V^+$, which is clearly lower than the equilibrium payoff of V^+. QED

Proof of Proposition 7: This is a straightforward extension of the proof of Proposition 6 above, adding the cost $c(q_I^1)^2$ to the equilibrium expected payoff, and $c(q^d)^2$ to each expected deviation payoff based on the required trading quantities q_I^1 and q^d. At $c = 0$, the equilibrium clearly exists based on the proof above. As c increases, the expected equilibrium payoff $\hat{\imath}V^+ - c(q_I^1)^2$ falls, eventually becoming negative, while the payoff to the "no trade" deviation $q^d = 0$ described above is not affected. Deviation therefore becomes profitable at some point as the payoff to choosing $q^d = 0$ (or possibly some other deviation) exceeds the equilibrium payoff. Any further increase in c can only further decrease the relative payoff of the equilibrium strategy. QED

Proof of Proposition 8: This follows almost directly from Proposition 7 and Proposition 4. From Proposition 4, an increase in ϵ reduces i^{*S} and therefore (weakly) reduces the equilibrium q_I^1 required to reach the stake i^{*S}, but does not change the potential deviation quantities q^d considered in the proof of Proposition 6. Thus, considering only the indirect effect of ϵ through its impact on q_I^1, the expected payoff of the equilibrium strategy increases relative to the expected payoffs of the deviations for any c as ϵ rises. Finally, note that there is also a direct effect of ϵ on the expected payoffs. However, it is straightforward to show that the equilibrium expected payoff increases in ϵ due to this direct effect faster than do the expected payoffs of the deviations holding q_I^1 and c fixed, which reinforces the mitigating effect on c^*. QED

Proof of Proposition 9: As noted in the text, it is straightforward to show that the full efficiency equilibria derived for the base model also exist without the agency problem, giving the weak part of the inequality. We prove the strict case by construction. Consider an equilibrium with $q_I^H = +5$, $q_I^M = +3$, and $q_I^L = -2$. Prices at equilibrium order flows with acceptance are as follows: $p(+7) = V_H$, $p(+5) = V_P^+$, $p(+3) = V_P^-$, and $p(+1) = V_M$, where V_P^+ reflects a $\frac{1}{2}$ chance of an H versus an M, and V_P^- reflects a $\frac{1}{5}$ probability of an H versus an M. All order flows below $+1$ have a price of 1 and D rejects. For out of equilibrium order flows $Q \in \{+2, +4\}$ we assume D accepts with beliefs leading to prices of V_P^+ and V_P^-, respectively. For higher out of equilibrium order flows we assume D accepts and the belief is that $\theta = H$, so the price is V_H, and for lower ones we assume the belief is that $\theta = L$ so D rejects and the price is 1.

First consider deviations by I after an H signal. It is straightforward to show that deviating to $+7$ is the most profitable upward deviation. Since D always accepts with or without deviation, we focus on expected trading profits. In the equilibrium they are $5(\frac{3}{4}V_H - \frac{1}{2}V_P^+ - \frac{1}{4}V_P^-)$. The deviation to $+7$ yields $7(\frac{1}{4}V_H - \frac{1}{4}V_P^+)$, which is clearly lower. A deviation down to $+3$ is similarly the best downward deviation. It yields an expected trading profit of $3(V_H - \frac{1}{4}V_P^+ - \frac{1}{2}V_P^- - \frac{1}{4}V_M)$, which again is clearly lower than the equilibrium payoff.

After an M, I will clearly never wish to deviate upward (the same acceptance probability but more trading losses). I's equilibrium expected payoff is $iV_M - 3(\frac{1}{4}V_P^+ + \frac{1}{2}V_P^- - \frac{3}{4}V_M)$. Consider a downward deviation to $+1$ (the best possible such deviation). This has an expected payoff of $i(\frac{1}{4} + \frac{3}{4}V_M) - \frac{1}{4}(V_P^- - V_M)$. Comparing this to the equilibrium payoff (and simplifying using the equalities $V_P^+ = \frac{1}{2}V_H + \frac{1}{2}V_M$, $V_P^- = \frac{1}{5}V_H + \frac{4}{5}V_M$, and $V_H = 3V_P - 2V_M$), deviation is profitable if $i < \frac{\frac{24}{5}(V_P - V_M)}{V_M - 1} < i^{*S}$.

Finally, consider deviations by I following an L. The equilibrium expected payoff is i. It is straightforward to show that the most profitable deviation is to -1, which yields $i(\frac{1}{4}V_L + \frac{3}{4}) + \frac{1}{4}(V_M - V_L)$. Comparing this to the equilibrium payoff, deviation is profitable if $i < \frac{V_M - V_L}{1 - V_L}$. The numerator and denominator are both weakly positive. As ϵ approaches its maximum, which is constrained to keep $V_L < 1$, the expression goes to infinity. As ϵ goes to zero, the numerator declines while the denominator rises, with a limiting value of 1 at $\epsilon = 0$, whereas i^{*S} goes to infinity as ϵ goes to zero. This suffices to prove the result. QED

Proof of Proposition 10: Directly calculating $\frac{\partial i^{*N}}{\partial \lambda}$ yields $\frac{8(2d-\epsilon)}{3\epsilon} > 0$. Directly calculating $\frac{\partial i^{*N}}{\partial d}$ yields $\frac{8(2\lambda - \frac{1}{2})}{3\epsilon} > 0$. Substituting κd for ϵ in i^{*N} yields $\frac{8(\lambda(2-\kappa)+\frac{1}{2}(\kappa-1))}{3}$, which is clearly independent of d. The proofs for i^{*S} are analogous. QED

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